Introduction to electron and photon beam physics

Zhirong Huang

SLAC and Stanford University

August 03, 2015
Lecture Plan

- Electron beams (1.5 hrs)
- Photon or radiation beams (1 hr)

References:

2. Helmut Wiedemann, Particle Accelerator Physics (Springer-Verlag, 2003).
4. David Attwood, *Soft X-rays and Extreme Ultraviolet Radiation (Cambridge, 1999)*
8. Images from various sources and web sites.
Electron beams

- Primer on special relativity and E&M
- Accelerating electrons
- Transporting electrons
- Beam emittance and optics
- Beam distribution function
Lorentz Transformation

\[
\begin{pmatrix}
 x \\
 y \\
 z \\
 ct
\end{pmatrix}
= 
\begin{pmatrix}
 1 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 \\
 0 & 0 & \gamma & \beta \gamma \\
 0 & 0 & \beta \gamma & \gamma
\end{pmatrix}
\begin{pmatrix}
 x^* \\
 y^* \\
 z^* \\
 ct^*
\end{pmatrix}
\]

\[
\beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \beta = \sqrt{1 - \frac{1}{\gamma^2}}
\]

\[
\gamma = \frac{E_e}{mc^2} = \frac{E_e[GeV]}{0.511 \times 10^{-3}} = 1957 E_e[GeV]
\]

\[
\beta = \sqrt{1 - \frac{1}{\gamma^2}} \approx 1 - \frac{1}{2\gamma^2},
\]

\[
1 - \beta \approx 5 \times 10^{-8} \text{ for } E_e = 1.5 \text{ GeV}
\]
Length Contraction and Time Dilation

Length contraction: an object of length $\Delta z^*$ aligned in the moving system with the $z^*$ axis will have the length $\Delta z$ in the lab frame

$$\Delta z = \frac{\Delta z^*}{\gamma}$$

Time dilation: Two events occurring in the moving system at the same point and separated by the time interval $\Delta t^*$ will be measured by the lab observers as separated by $\Delta t$

$$\Delta t = \gamma \Delta t^*$$
Energy, Mass, Momentum

Energy

\[ E = T + mc^2 \]

Kinetic energy \quad Rest mass energy

Electrons rest mass energy 511 keV (938 MeV for protons), 1 eV = 1.6 \times 10^{-19} \text{ Joule}

Momentum

\[ p = \gamma \beta mc \]

Energy and momentum

\[ E^2 = p^2 c^2 + m^2 c^4, \quad E = \gamma mc^2. \]
Relativistic acceleration

Momentum change

\[ \frac{dp}{dt} = m \gamma \frac{dv}{dt} + m v \frac{d\gamma}{dt}. \]

With

\[ \frac{d\gamma}{dt} = \frac{d}{d\beta} \frac{1}{\sqrt{1 - \beta^2}} \frac{d\beta}{dt} = \gamma^3 \frac{\beta}{c} \frac{dv}{dt} \]

we get the equation of motion

\[ F = \frac{dp}{dt} = m \left( \gamma \frac{dv}{dt} + \gamma^3 \frac{\beta}{c} \frac{dv}{dt} v \right). \]

For a force parallel to the particle propagation \( v \) we have \( \dot{v}v = \dot{v}v \) and

\[ \frac{dp}{dt} = m \gamma \left( 1 + \gamma^2 \beta \frac{v}{c} \right) \frac{dv}{dt} = m \gamma^3 \frac{dv}{dt}. \]

On the other hand, if the force is directed normal to the particle propagation we have \( \dot{v} = 0 \) and (1.18) reduces to

\[ \frac{dp}{dt} = m \gamma \frac{dv}{dt}. \]

Beam dynamics drastically different for parallel and perpendicular acceleration!

Negligible radiation for parallel acceleration at high energy.
Maxwell’s Equations

\[ \nabla \cdot D = \rho \]
\[ \nabla \cdot B = 0 \]
\[ \nabla \times E = -\frac{\partial B}{\partial t} \]
\[ \nabla \times H = j + \frac{\partial D}{\partial t} \]

**D** = \( \varepsilon_0 E \)

**B** = \( \mu_0 H \)

\[ c = (\varepsilon_0 \mu_0)^{-1/2} \]

\[ Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377 \text{ Ohm} \]

- Wave equation

\[ \left( \frac{\partial^2}{\partial t^2} - c^2 \nabla^2 \right) E = -\frac{1}{\varepsilon_0} \left( \frac{\partial j}{\partial t} + c^2 \nabla \rho \right) \]

- Lorentz transformation of fields

\[ E_z = E'_z, \quad E_\perp = \gamma \left( E'_\perp - \mathbf{v} \times \mathbf{B}' \right) , \]
\[ B_z = B'_z, \quad B_\perp = \gamma \left( B'_\perp + \frac{1}{c^2} \mathbf{v} \times \mathbf{E}' \right) \]
Field of a moving electron

- In electron’s frame, Coulomb field is
  \[ E' = \frac{1}{4\pi\varepsilon_0} \frac{e r'}{r'^3} \]

- In lab frame, space charge fields are
  \[ E_x = \frac{1}{4\pi\varepsilon_0} \frac{e\gamma x}{[x^2 + y^2 + \gamma^2(z - vt)^2]^{3/2}} \]
  \[ E_y = \frac{1}{4\pi\varepsilon_0} \frac{e\gamma y}{[x^2 + y^2 + \gamma^2(z - vt)^2]^{3/2}} \]
  \[ E_z = \frac{1}{4\pi\varepsilon_0} \frac{e\gamma(z - vt)}{[x^2 + y^2 + \gamma^2(z - vt)^2]^{3/2}} \]
  \[ B = \frac{1}{c^2} v \times E \]
Lorentz Force

Lorentz force

\[ F = eE + ev \times B \]

Momentum and energy change

\[ \Delta p = \int F \, dt \]
\[ \Delta E = \int F \, ds \quad ds = \mathbf{v} \, dt \]

Energy exchange through \( E \) field only

\[ \Delta E = \int F \, ds = e \int E \cdot ds + e \int (v \times B) \cdot \mathbf{v} \, dt \]

= 0

No work done by magnetic field!
Guiding beams: dipole

- Lorentz force

\[ F = eE + e\mathbf{v} \times \mathbf{B} \]

- Centrifugal force

\[ F_{cf} = \frac{\gamma mc^2 \beta^2}{\rho} \]

- Bending radius is obtained by balance the forces

\[ \frac{1}{\rho} = \frac{eB}{\gamma \beta mc^2} \]

\[ \frac{1}{\rho} \left[ \text{m}^{-1} \right] = 0.2998 \frac{B[\text{T}]}{\beta E[\text{GeV}]} \]
Cyclotron

If beam moves circularly, re-traverses the same accelerating section again and again, we can accelerate the beam repetitively.

Ernest O. Lawrence in 1930

The first cyclotron with a diameter of 5 inches

[Ref.]: Photography gallery of Lawrence Berkeley National Laboratory, http://cso.lbl.gov/photo/gallery/
Lawrence started to construct a cyclotron, as the machine later was named, in early 1930. A graduate student, M. Stanley Livingston, did much of the work of translating the idea into working hardware. In January 1931 Lawrence and Livingston met their first success. A device about 4.5 inches in diameter used a potential of 1,800 volts to accelerate hydrogen ions up to energies of 80,000 electron volts. Lawrence immediately started planning for a bigger machine. In summer 1931 an eleven-inch cyclotron achieved a million volts.

"Dr Livingston has asked me to advise you that he has obtained 1,100,000 volt protons. He also suggested that I add ‘Whoopee’!"

—Telegram to Lawrence, http://www.aip.org/history/lawrence/first.htm 3 August 1931

Lawrence was my teacher when I built the first cyclotron. He got a Nobel prize for it. I got a Ph.D. (- S. Livingston, years later)
Cyclotron does not work for relativistic beams.

- huge dipole, compact design, 
  $B = \text{constant}$
  low energy, single pass.

- varying $B$, small magnets, high energy
Synchrotron

GE synchrotron observed first synchrotron radiation (1946) and opened a new era of accelerator-based light sources.

The first purpose-built synchrotron to operate was built with a glass vacuum chamber.
Electron linac

The rf energy is used to launch a traveling wave or standing wave in an array of cavities.

The linac structure is designed such that the phase of the EM wave is synchronized with the beam, i.e. \( v_p \sim c \). In a smooth waveguide, the phase velocity \( v_p > c \). Those disks are used to slow down the waveguide phase velocity in order to achieve synchronism with the electron beams.

\[ V = K \sqrt{P_0 \ell r_s} , \text{ where } K < 1 \]
Electron Linac (disk loaded structure)

[toward the right] +

Electric Field

[toward the left] -

Positive particles

Position

(amount of energy boost)

Negative particles

behind the bunch

on time

ahead

ahead

on time

behind

1/20,000,000,000 second later
(notice how far the bunches have moved)

[Ref.] http://www.slac.stanford.edu
We have accelerated electrons.
SLAC linac

35 Total 12-m CryoModules for LCLS-II
Stanford Linear Accelerator Center (SLAC)

50 × 50 GeV e-e+
Livingston Plot for High-Energy Accelerators

The diagram illustrates the historical development of particle accelerators, showing the progression of particle energy over the years. It highlights various types of accelerators and their commissioning years, with notable milestones such as the LHC and ILC. The timeline spans from 1930 to 2010, with specific emphasis on the evolution of high-energy physics in the 20th century.
**Linac Coherent Light Source (LCLS) at SLAC**

X-FEL based on last 1-km of existing 3-km linac

Proposed by C. Pellegrini in 1992

1.5-15 Å

(14-4.3 GeV)

Injector (35°) at 2-km point

Existing 1/3 Linac (1 km) (with modifications)

New $e^-$ Transfer Line (340 m)

X-ray Transport Line (200 m)

Undulator (130 m)

Near Experiment Hall

Far Experiment Hall
Beam description

Beam phase space \((x, x', y, y', \Delta t, \Delta \gamma)\)

\[
x' \equiv \frac{dx}{dz} = \frac{dx/dt}{dz/dt} = \frac{1}{v_z} \frac{dx}{dt}
\]

\[
\Delta \gamma_j \equiv \gamma_j - \gamma_0
\]

Consider paraxial beams such that

\[
|x'| = \sqrt{x'^2 + y'^2} \approx \frac{1}{c} \sqrt{v_x^2 + v_y^2} \ll 1
\]
**Linear optics for beam transport**

- **Transport matrix**

\[
\begin{bmatrix}
x \\
x'
\end{bmatrix}_o = M(z_i, z_o)
\begin{bmatrix}
x \\
x'
\end{bmatrix}_i
\]

- **Free space drift**

\[
\begin{bmatrix}
x \\
x'
\end{bmatrix}_o = \begin{bmatrix} 1 & \ell \\ 0 & 1 \end{bmatrix}
\begin{bmatrix}
x \\
x'
\end{bmatrix}_i \equiv M_\ell
\begin{bmatrix}
x \\
x'
\end{bmatrix}_i
\]

- **Quadrupole (de-)focusing**

\[
\begin{bmatrix}
x \\
x' \\
y \\
y'
\end{bmatrix}_o = M_f
\begin{bmatrix}
x \\
x' \\
y \\
y'
\end{bmatrix}_i = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/f & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 1 \end{bmatrix}
\begin{bmatrix}
x \\
x' \\
y \\
y'
\end{bmatrix}_i
\]
Beam properties

- Second moments of beam distribution

\[ \sigma_x^2(z) = \langle x^2 \rangle = \frac{1}{N_e} \sum_j x_j^2. \]

\[ \sigma_{x'}^2(z) = \langle x'^2 \rangle = \frac{1}{N_e} \sum_j x'_j^2. \]

\[ \langle x x' \rangle = \frac{1}{N_e} \sum_j x_j x'_j. \]

- rms size

- rms divergence

- correlation

\[ \langle x x' \rangle = 0 \]

\[ \langle x x' \rangle = 0 \]

\[ \langle x x' \rangle \neq 0 \]
**Beam emittance**

- **Emittance or geometric emittance**
  \[ \varepsilon_x \equiv \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle} \]

- Emittance is **conserved** in a **linear** transport system.

- **Normalized emittance** is conserved in a linear system including acceleration.
  \[ \varepsilon_{x,n} = \beta_z \gamma \varepsilon_x \approx \gamma \varepsilon_x \]

- Normalized emittance is hence an important figure of merit for electron sources.

- Preservation of emittances is critical for accelerator designs.
Beam optics function

**Optics functions (Twiss parameters)**

\[
\beta_x = \frac{\langle x^2 \rangle}{\varepsilon_x} \quad \gamma_x = \frac{\langle x'^2 \rangle}{\varepsilon_x} \quad \alpha_x = -\frac{\langle xx' \rangle}{\varepsilon_x}
\]

\[
\beta_x \gamma_x - \alpha_x^2 = 1
\]

**Given beta function along beamline**

\[
\sigma_x(z) = \sqrt{\varepsilon_x \beta_x(z)}
\]
Free space propagation

- Single particle

\[
\begin{bmatrix} x \\ x' \end{bmatrix}_o = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_i
\]

- Beam envelope

\[
\langle x_o^2 \rangle = \langle (x_i + z x'_i)^2 \rangle = \langle x_i^2 \rangle + 2z \langle x_i x'_i \rangle + z^2 \langle x'_i^2 \rangle
\]

\[
\beta_x(z) = \beta_x(0) + z^2 \gamma_x(0)
\]

\[
\sigma_x(z) = \sqrt{\varepsilon_x \beta_x(z)} = \sqrt{\varepsilon_x \left( \beta_x^* + \frac{z^2}{\beta_x^*} \right)}
\]

- Analogous with Gaussian laser beam

\[
\varepsilon_x \leftrightarrow \frac{\lambda}{4\pi}, \quad \beta_x^* \leftrightarrow Z_R.
\]
FODO lattice

- Multiple elements (e.g., FODO lattice)

\[ M = M_N M_{N-1} \ldots M_2 M_1 \]

\[
M_{\text{FODO}} = \begin{bmatrix}
1 & 0 \\
-1/2f & 1
\end{bmatrix}
\begin{bmatrix}
1 & \ell \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
1/f & 1
\end{bmatrix}
\begin{bmatrix}
1 & \ell \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
-1/2f & 1
\end{bmatrix}

= \begin{bmatrix}
1 - \frac{\ell^2}{2f^2} & 2\ell \left(1 + \frac{\ell}{2f}\right) \\
-\frac{\ell}{2f^2} \left(1 - \frac{\ell}{2f}\right) & 1 - \frac{\ell^2}{2f^2}
\end{bmatrix}.
\]
For periodic motion we have $\beta_x(0) = \beta_x(2\ell)$ and $\gamma_x(0) = \gamma_x(2\ell)$, while vanishing correlation $\alpha_x$ at the two planes implies that $\beta_x(0) = 1/\gamma_x(0)$.

- **Maximum beta**

\[\beta_x(0) = 2 \sqrt{\frac{2f^3 + f^2\ell}{2f - \ell}} \approx 2 |f| \left(1 + \frac{\ell}{2f}\right)\]

- **Minimum beta**

\[\beta_x(\ell) \approx 2 |f| \left(1 - \frac{\ell}{2f}\right)\]

- **When $f >> l$**

\[\beta_x(z) \approx \bar{\beta}_x = 2f \quad \rightarrow \quad \langle x^2 \rangle \approx 2\varepsilon_x f\]

\[\gamma_x(z) \approx \frac{2}{\bar{\beta}_x} = \frac{1}{f} \quad \rightarrow \quad \langle x'^2 \rangle \approx \frac{\varepsilon_x}{f}\]

\[\alpha_x^2(z) \approx \bar{\beta}_x \bar{\gamma}_x - 1 = 1 \quad \rightarrow \quad \langle xx' \rangle \approx \pm \varepsilon_x.\]
Electron distribution in phase space

- We define the distribution function $F$ so that

$$N_e F(\Delta t, \Delta \gamma, x, x'; z) \, dx \, dx' \, d(\Delta t) \, d(\Delta \gamma)$$

is the number of electrons per unit phase space volume.

- Since the number of electrons is an invariant function of $z$, distribution function satisfies **Liouville theorem**

$$\frac{d}{dz} \frac{dF}{dz} = \left[ \frac{\partial}{\partial z} + (\Delta t)' \frac{\partial}{\partial \Delta t} + (\Delta \gamma)' \frac{\partial}{\partial \Delta \gamma} + x' \cdot \frac{\partial}{\partial x} + x'' \cdot \frac{\partial}{\partial x'} \right] F = 0$$
Gaussian beam distribution

- Represent the ensemble of electrons with a continuous distribution function (e.g., Gaussian in $x$ and $x'$)

$$F(x, x'; z) = \frac{1}{2\pi \varepsilon_x} \exp\left\{-\frac{1}{2\varepsilon_x} \left[ \gamma_x(z) x^2 + \beta_x(z) x'^2 + 2\alpha_x(z) xx' \right] \right\}$$

- For free space propagation

$$\beta_x(z) = \beta_x^* + \frac{z^2}{\beta_x^*}$$

$$F(x, x'; z) = \frac{1}{2\pi \varepsilon_x} \exp\left[ -\frac{(x - x' z)^2}{2\beta_x^* \varepsilon_x} - \frac{x'^2}{2\varepsilon_x / \beta_x^*} \right]$$

- Distribution in physical space can be obtained by integrating $F$ over the angle

$$\int dx' F(x, x'; z) = \exp\left[ -\frac{x^2}{2\sigma_x^* \left( 1 + z^2 / \beta_x^* \right)} \right] \sqrt{\frac{2\pi \sigma_x^*}{\sqrt{1 + z^2 / \beta_x^*}}}$$
Photon or radiation beams

- Introduction to radiation
- Radiation diffraction and emittance
- Coherence and Brightness
- Radiation intensity and bunching
- Accelerator based light sources
Photon wavelength and energy

\[ \hbar \omega \cdot \lambda = hc = 1239.842 \text{ eV nm} \]

- See smaller features
- Write smaller patterns
- Elemental and chemical sensitivity
**Opportunities for Tunable Source of Radiation**

- **Single pass FELs (SASE or seeded)**
- **Synchrotron radiation**
- **Undulator radiation**

### Various accelerator and non-acc. sources

- **FEL oscillators (High-average power)**

### Graph

- **Peak Power (W)**
  - $10^{13}$
  - $10^9$
  - $10^7$
  - $10^5$
  - $10^3$
  - 1

- **Wavelength (nm)**
  - 10 mm
  - 1 mm
  - 100 μ
  - 10 μ
  - 1 μ
  - 100 nm
  - 10 nm
  - 1 nm
  - 1 Å
Radiation from Accelerated Electrons

The field outside of the circle of radius $ct$ “does not know” that the charge has been moved.

If the charge was moved twice, then the field lines at time $t > t_1$ would look like this—there will be two spheres, with the radiation layers between them.
Three forms of synchrotron radiation

1. Bending magnet radiation
2. Wiggler radiation
3. Undulator radiation
Shintake Radiation Demo Program
Radiation propagation and diffraction

Wave propagation in free space

\[
\left[ \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} + k^2 \right] E_\omega(x; z) = 0, \quad k = \frac{\omega}{c} = \frac{2\pi}{\lambda}
\]

Angular representation

\[
\mathcal{E}_\omega(\phi; z) = \frac{1}{\lambda^2} \int d\mathbf{x} \, e^{-ik\phi \cdot \mathbf{x}} E_\omega(\mathbf{x}; z)
\]

\[
E_\omega(x; z) = \int d\phi \, e^{ik\phi \cdot x} \mathcal{E}_\omega(\phi; z).
\]

General solution

\[
E_\omega(x; z) = \int d\phi \, \exp \left[ ik(\phi \cdot x \pm z\sqrt{1 - \phi^2}) \right] \mathcal{E}_\omega(\phi; 0)
\]

Paraxial approximation \((\phi^2 \ll 1)\)

\[
\mathcal{E}_\omega(\phi; z) = e^{ik(1-\phi^2/2)z} \mathcal{E}_\omega(\phi; 0)
\]
Gaussian beam and radiation emittance

Single electron radiation can be approximated by Gaussian beam \( \Rightarrow \) Gaussian fundamental mode at waist \( z=0 \)

\[
E(x; 0) = E_0 \exp \left( -\frac{x^2}{4\sigma_r^2} \right)
\]

\[
\mathcal{E}(\phi; 0) = \mathcal{E}_0 \exp \left( -\frac{\phi^2}{4\sigma_{r'}^2} \right)
\]

\[
\sigma_r \sigma_{r'} = \frac{\lambda}{4\pi} \equiv \varepsilon_r
\]

At arbitrary \( z \)

\[
E(x; z) = \frac{E_0}{\sqrt{1 + i\sigma_{r'} z / \sigma_r}} \exp \left[ -\frac{x^2}{4\sigma_r^2 (1 + i\sigma_{r'} z / \sigma_r)} \right]
\]

\[
= \frac{E_0}{(1 + z^2 / Z_R^2)^{1/4}} \exp \left[ -\frac{x^2 (1 - i z / Z_R)}{4\sigma_r^2 (1 + z^2 / Z_R^2)} - \frac{i}{2} \tan^{-1} \left( \frac{z}{Z_R} \right) \right]
\]

\[
\sigma_r(z) = \sqrt{\frac{\lambda}{4\pi} \left( Z_R + \frac{z^2}{Z_R^2} \right)}
\]

\[
Z_R \equiv \sigma_r / \sigma_{r'} = 2 \kappa \sigma_r^2
\]

Analogous with electron beam

\[
\varepsilon_x \leftrightarrow \frac{\lambda}{4\pi} \quad \beta_x^* \leftrightarrow_{40} Z_R.
\]
Coherence

$E(x,t)$ at location $z$

longitudinal coherence

transverse coherence

wavefronts (i.e. lines of equal phase)

direction of propagation $z$

R. Ischebeck
Transverse (Spatial) Coherence

- Transverse coherence can be measured via the interference pattern in Young's double slit experiment.
- Near the center of screen, transverse coherence determines fringe visibility

Persistence of fringes as the source grows from a point source to finite size.

\[ d \cdot 2\theta_{\text{FWHM}} \approx \frac{\lambda}{2} \]
Phase space criteria for transverse coherence

- Initial phase space area $4\pi R \gg \lambda$
- Final phase space area $4Ra/D \lesssim \lambda/2$
- Coherent flux is reduced by $M_T$
- Show this criteria from physical optics argument
Define a coherence length $\ell_{\text{coh}}$ as the distance of propagation over which radiation of spectral width $\Delta \lambda$ becomes $180^\circ$ out of phase. For a wavelength $\lambda$ propagating through $N$ cycles

$$\ell_{\text{coh}} = N\lambda$$

and for a wavelength $\lambda + \Delta \lambda$, a half cycle less ($N - \frac{1}{2}$)

$$\ell_{\text{coh}} = (N - \frac{1}{2}) (\lambda + \Delta \lambda)$$

Equating the two

$$N = \frac{\lambda}{2\Delta \lambda}$$

so that

$$\ell_{\text{coh}} = \frac{\lambda^2}{2\Delta \lambda}$$
Chaotic light

- Radiation from many random emitters (Sun, SR, SASE FEL)

\[ E(t) = \sum_{j=1}^{N_e} E_0(t - t_j) = e_0 \sum_{j=1}^{N_e} \exp \left[ -\frac{(t - t_j)^2}{4\sigma^2} - i\omega_1(t - t_j) \right] \]

- Correlation function and coherence time

\[ C(\tau) \equiv \frac{\langle \int dt \ E(t)E^*(t + \tau) \rangle}{\langle \int dt \ |E(t)|^2 \rangle} \]

\[ t_{coh} \equiv \int dt \ |C(\tau)|^2 \]
Temporal mode and fluctuation

- Number of regular temporal regions is # of coherent modes

\[ M_L \approx \frac{T}{t_{coh}} = \frac{T}{2\sqrt{2\pi \sigma_T}} \approx \frac{T}{5\sigma_T}. \]

- Intensity fluctuation

\[ \frac{\Delta W}{W} = \frac{1}{\sqrt{M_L}}. \]

- Same numbers of mode in frequency domain

\[ E_\omega = \frac{e_0 \sigma_T}{\sqrt{\pi}} \sum_{j=1}^{N_e} \exp \left[ -\frac{(\omega - \omega_1)^2}{4\sigma_\omega^2} + i\omega t_j \right] \]

- Fourier limit, minimum longitudinal phase space

\[ c\sigma_T \cdot \frac{\sigma_\omega}{\omega_1} = \frac{\lambda_1}{4\pi}, \]

- Longitudinal phase space is \( M_L \) larger than Fourier limit

- Total # of modes

\[ M = M_L M_T^2. \]
Light Bulb vs. Laser

Radiation emitted from light bulb is chaotic.

Pinhole can be used to obtain spatial coherence.

Monochromator can be used to obtain temporal coherence.

Pinhole and Monochromator can be combined for coherence.

Laser light is spatially and temporally coherent.

A. Schawlow (Nobel prize on laser spectroscopy), Scientific Americans, 1968
Brightness

\[ B = \frac{\text{Photons in unit spectral range in unit time}}{(\text{source size} \times \text{divergence})^2} \]

Units: photons/s/mm²/mrad²/0.1%BW

Peak

Average
Incoherent radiation from many electrons

Such a beam can be described by the convolution of the coherent Gaussian beam with the electron distribution in phase space.

Effective source size and divergence

\[ \Sigma_x = \sqrt{\sigma_x^2 + \sigma_r^2} \quad \Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{r'}^2} \]

When electron beam emittance \( \sigma_x \sigma_{x'} \gg \lambda/(4\pi) \)

\[ \Sigma_x \Sigma_{x'} \gg \frac{\lambda}{4\pi} \]

\# of transverse modes

\[ M_T = \frac{\Sigma_x \Sigma_{x'}}{\lambda/4\pi} = \frac{\varepsilon_x}{\varepsilon_r} \]
Evolution of X-ray Light Sources

- GE synchrotron (1946) opened a new era of accelerator-based light sources.

- These light sources have evolved rapidly over four generations.
  - The first three-generations are based on synchrotron radiation.
  - The forth-generation light source is a game-changer based on FELs.

- The dramatic improvement of brightness and coherence over 60 years easily outran Moore’s law.
State-of-art storage rings have **pulse duration** $\sim 10$ ps, **emittance** $\sim 1$ nm.

Diffraction-limited storage rings with **emittance** $\sim 10$ pm are under active R&D.
Radiation intensity

What if emitters are not random in time

\[
\langle |E(\omega)|^2 \rangle = |E^0_\omega|^2 \left( \sum_{j=1}^{N_e} e^{i\omega t_j} \right)^2
\]

\[
\left( \sum_{j=1}^{N_e} e^{i\omega t_j} \right)^2 = N_e + \sum_{j \neq k}^{N_e} e^{i\omega (t_j - t_k)}
\]

\[
\left( \sum_{j \neq k}^{N_e} e^{i\omega (t_j - t_k)} \right)^2 = N_e (N_e - 1) \left| \int dt \, f(t) e^{i\omega t} \right|^2
\]

For an electron bunch with rms bunch length \( \sigma_e \)

\[
f(t) = \frac{1}{\sqrt{2\pi} \sigma_e} \exp \left( -\frac{t^2}{2\sigma_e^2} \right)
\]

\[
\langle |E(\omega)|^2 \rangle = N_e |E^0_\omega|^2 \left[ 1 + (N_e - 1) e^{-\omega^2 \sigma_e^2} \right]
\]

When intensity from many electrons add \text{incoherently} \((\sim N_e)\)
Bunching and coherent radiation

- If the bunch length is shorter than the radiation wavelength
  \[(N_e - 1)e^{-\omega^2 \sigma^2_e} \geq 1\]

\[\left\langle |E(\omega)|^2 \right\rangle = N_e |E_\omega^0|^2 \left(1 + (N_e - 1)|f(\omega)|^2 \right)\]

- Radiation intensity from many electrons add **coherently** \((\sim N_e^2)\)

- Another way to produce bunching from a relatively long bunch is through so-called microbunching

![Graphs showing bunching and coherent radiation](image)