S3EPB
SLAC Summer Seminar on Electron and Photon Beams

Electron Beam Diagnostics

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U.S. DEPARTMENT OF ENERGY
Office of Science

SLAC NATIONAL ACCELERATOR LABORATORY

Electron beam diagnostics for high brightness beams

Light sources driven by **high brightness** electron beams:

- **Dense phase space** distributions to drive coherent X-ray generation

$$B = \frac{Q}{\varepsilon_{x,n}\varepsilon_{y,n}\varepsilon_{z,n}}$$

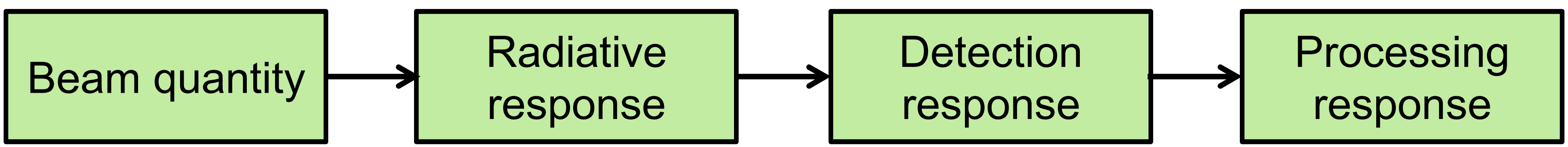
- Synchrotrons/FELs have *tiny* beams
  - $\varepsilon_{x/y,n} < 1$ mm-mrad → 10s μm spot sizes
  - *Transverse coherence*
- Also ultrashort, well-defined energy
  - Synchrotrons order picoseconds, typ.
  - FELs order *femtoseconds*, longitudinal coherence enhancement

Diagnostics: A recurring theme

Measuring any beam quantity confounded by responses

General scheme is:

1. Beam creates some EM field related to what we want to measure
2. Some device or pick up converts EM field to electrical signal(s)
3. Electrical signal gets processed / converted to meaningful number



Every step has some transient (or equivalently, spectral) response

Task is to preserve a signal proportional to the quantity of interest

Electron beam diagnostics for high brightness beams

- Where is the beam?

- How wide is it?

- How short is it?

Electron beam diagnostics for high brightness beams

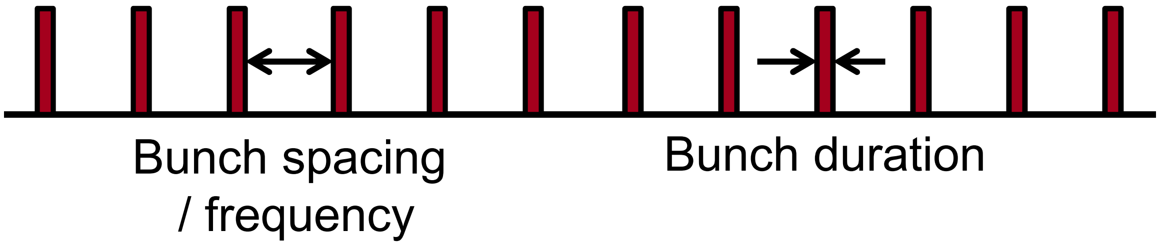
- Beam location
  - Charge and position measurement devices
- Transverse space / shape and emittance
  - Screen monitors
  - Wire scanners
- Longitudinal *t* profile
  - Frequency domain techniques
  - Time domain techniques

Beam current / bunch charge montitors

Which quantity to measure?

- Bunch charge monitor: Single bunch measurement (fast)
- Beam current monitor: Average charge per unit time (slow)

Consider beam pattern

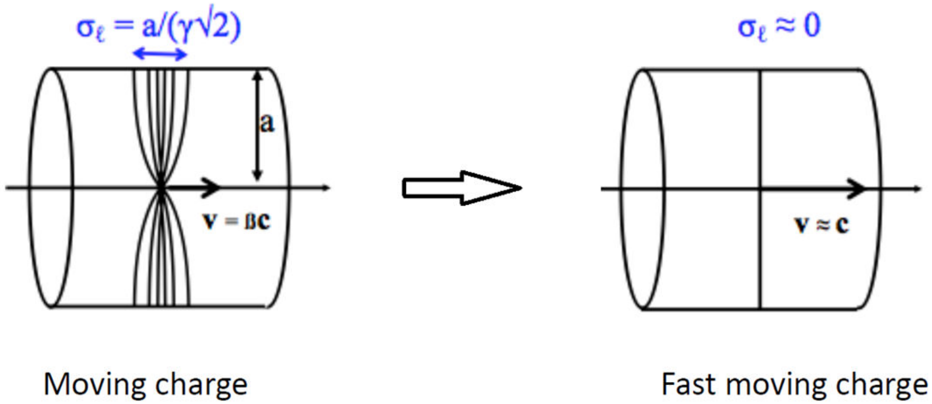


Examples

- APS ring: 80 ps bunches @ 6.5 MHz
- LCLS: 50 fs @ 120 Hz

Beam current / bunch charge montitors

“Radiative response” = Bunch velocity field



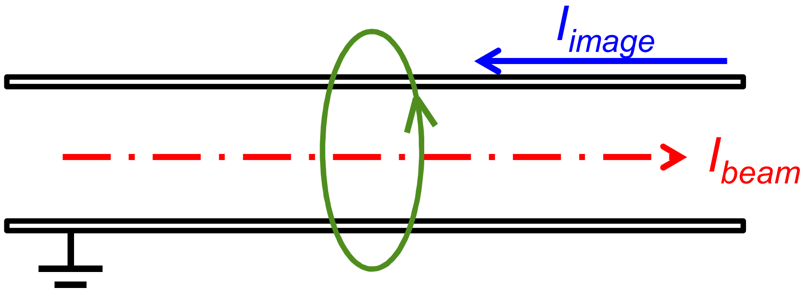
Relativistic single electron generates Lorentz-contracted Coulomb field

- Radial E-field (and azimu. B) opening angle inv. prop. to γ
- Example: 1 GeV beam in 50 mm OD pipe → **60 fs** rise/fall at wall
- For longer (ps) bunches, single bunch profile could be resolved
- To MHz-GHz electronics, can look like broadband, $\delta(t)$ impulse

Ref [2]

Beam current / bunch charge montitors

Beam image / wall currents:



Ampere's Law outside beam pipe, $\int \boldsymbol{H \cdot dl} = I$

- If outside beam pipe, $I_{image} = I_{beam}$, so $H = 0$
- Wall not *perfect* conductor, shields at rate of skin depth with resistivity ρ at freq. f $\delta = \frac{10^3}{2\pi}\sqrt{\frac{10\rho}{f}}$
- In MHz range, strong attenuation

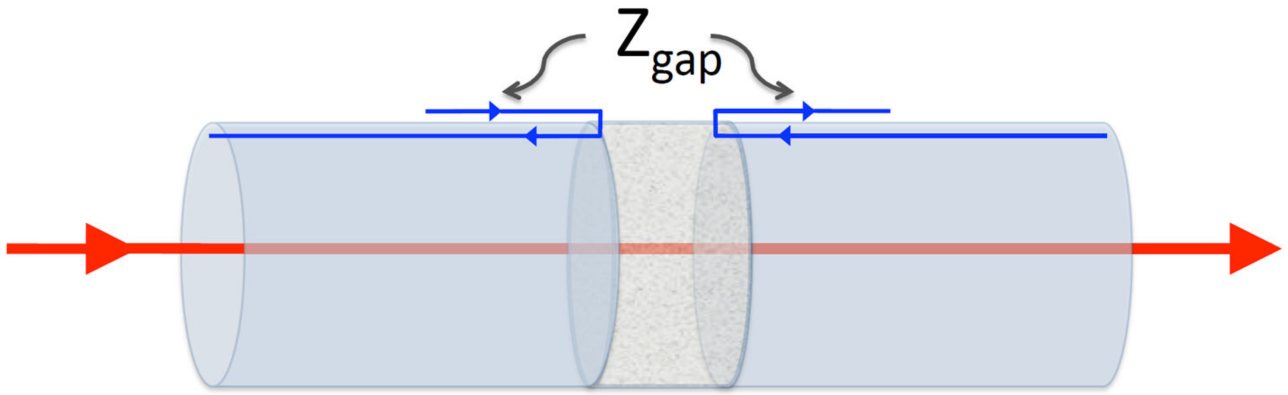
Skin depth (mm)	**1 KHz**	**10 KHz**	**100 KHz**	**1 MHz**	**10 MHz**
Copper	2.1	0.66	0.21	0.066	0.021
302 Stainless Steel	13.3	4.2	1.3	0.42	0.13

Ref [2]

Beam current / bunch charge montitors

No field outside beam pipe, two options…

1. Install a detector in the pipe(/vacuum)
2. Insert a ceramic break
   - Forces image current to find another path, and it will

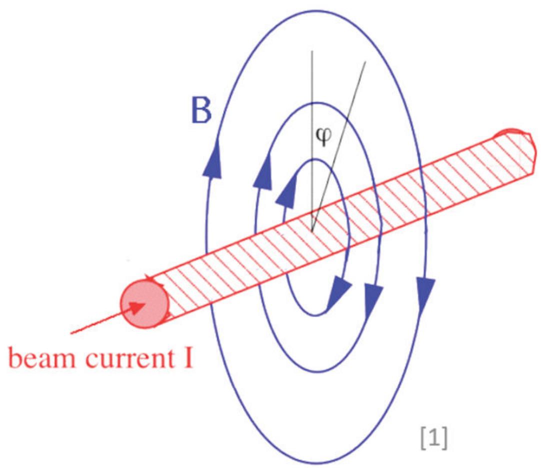


Ref [2]

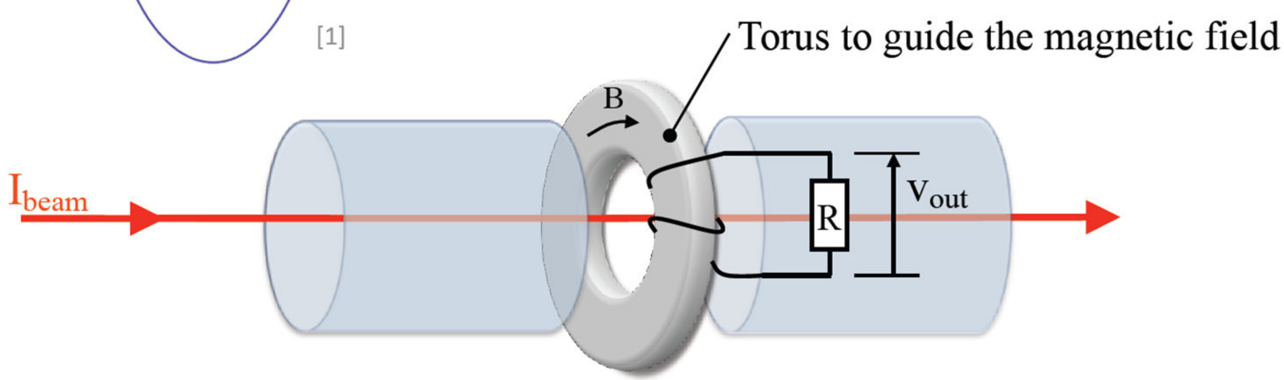
Integrating current transformer

A detector option: Probe field through beam magnetic field

- Non-destructive

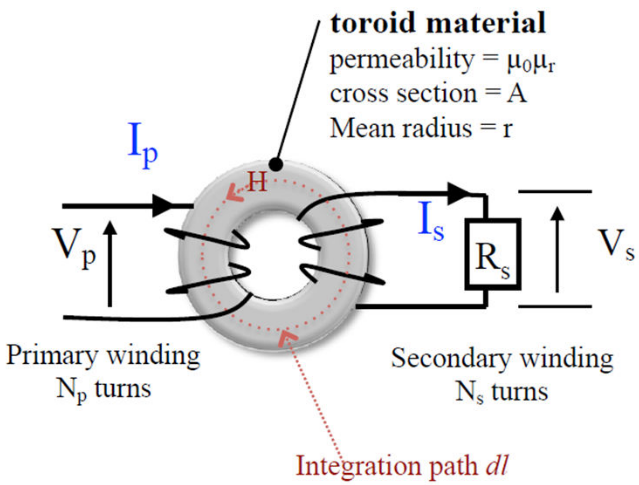


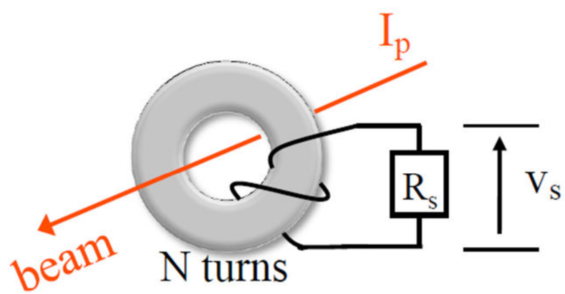
$$\text{Line current field:}\quad \vec{B} = \mu \cdot \frac{I_{beam}}{2\pi r}\,\vec{a}_{\varphi}$$



Ref [2]

Integrating current transformer





Ampere's Law:

$$\oint H \cdot dl = N_p I_p + N_s I_s = I_p + N_s I_s \quad \text{with } N_p = 1 \quad \Rightarrow$$

$$H = (I_p + N_s I_s)/2\pi r \qquad (1)$$

Flux: (thin toroid approximation)

$$\Phi = \int_S B\, dS = \mu H A = \mu A (I_p + N_s I_s)/2\pi r \quad \text{with } A \text{ as area} \qquad (2)$$

Faraday's Law:

$$V_s = -N_s \cdot \frac{d\Phi}{dt} = I_s \cdot R_s \qquad (3)$$

Combine (2) and (3):

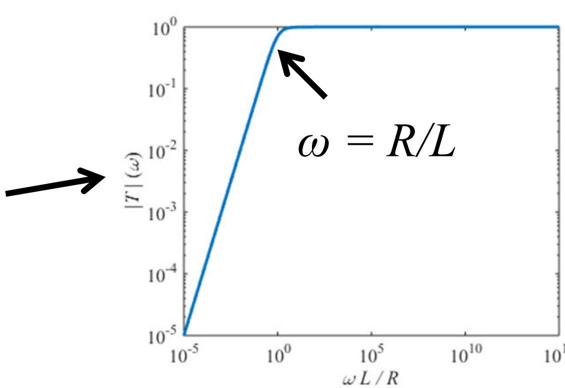
$$I_s \cdot R_s = -N_s \cdot \frac{\mu A}{2\pi r} \cdot \frac{d(I_p + N_s I_s)}{dt} \quad \text{with} \quad L_s = \frac{N_s^2 \mu A}{2\pi r} \Rightarrow$$

Differential equation:

$$\frac{dI_s}{dt} + \frac{R_s}{L_s} I_s = -\frac{1}{N_s} \cdot \frac{dI_p}{dt}$$

Laplace rewrite:

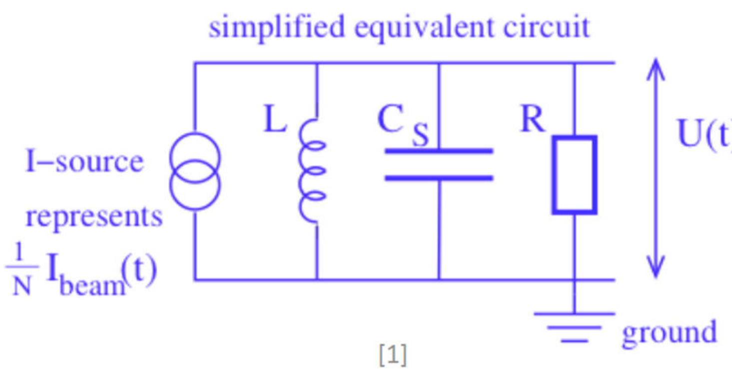
$$\frac{I_s(i\omega)}{I_p(i\omega)} = -\frac{1}{N_s} \cdot \frac{i\omega}{(i\omega + R_s/L_s)}$$



Ref [2]

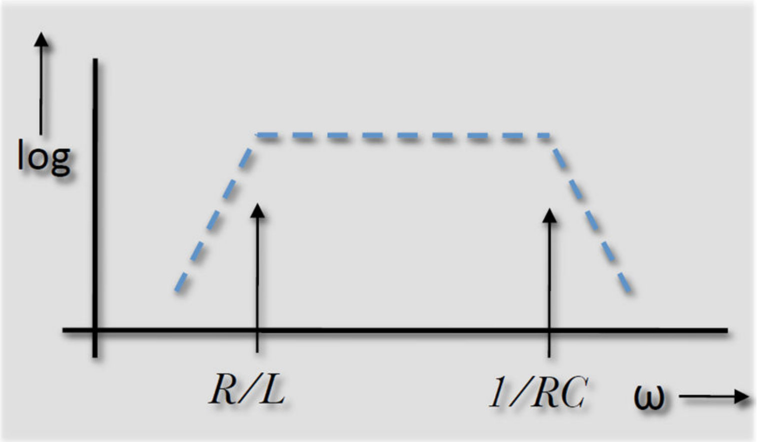
Integrating current transformer

But ah, there is also capcitance: Actually an RLC circuit



$$\frac{1}{Z} = \frac{1}{R} + \frac{1}{i\omega L} + i\omega C \Rightarrow$$

$$Z = \frac{i\omega L}{1 + i\omega L/R - (\omega L/R)\cdot(\omega RC)}$$



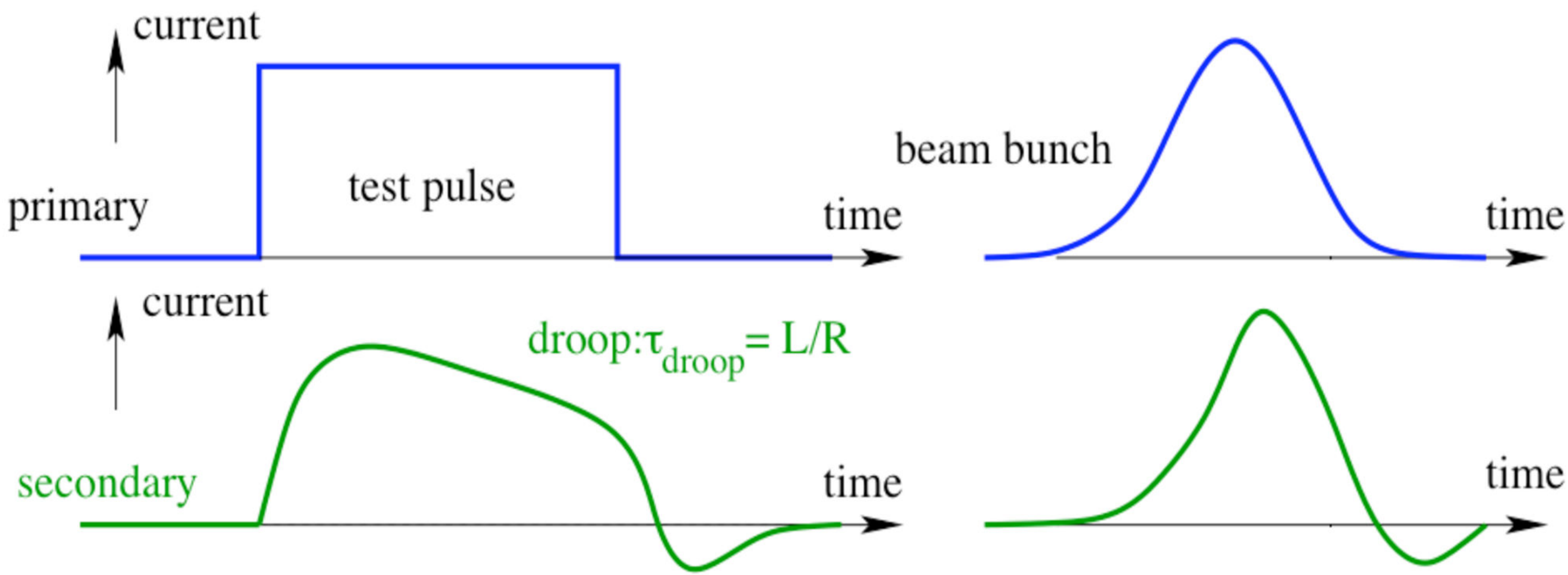
A band-pass circuit, will impose limit on the rise and fall of signal

- Rise time = *RC*
- Fall time = *L / R*

Ref [2]

Pulse distortion

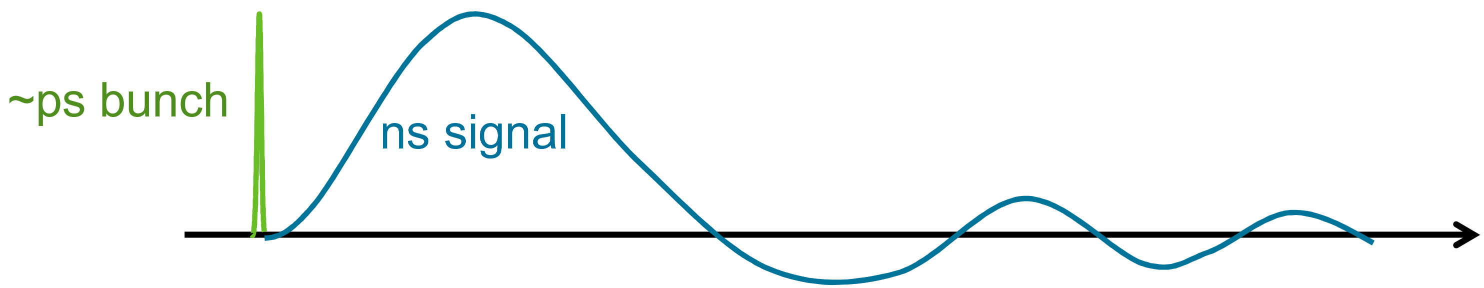
Finite rise time from *RC*, signal droop due to *L/R:*

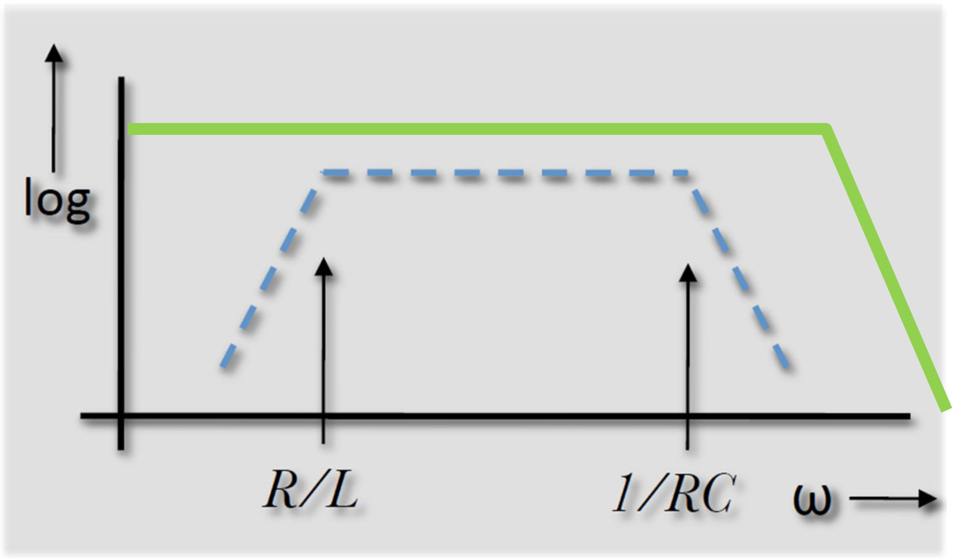


Pulse distortion

Ultra-short bunches = impulse response

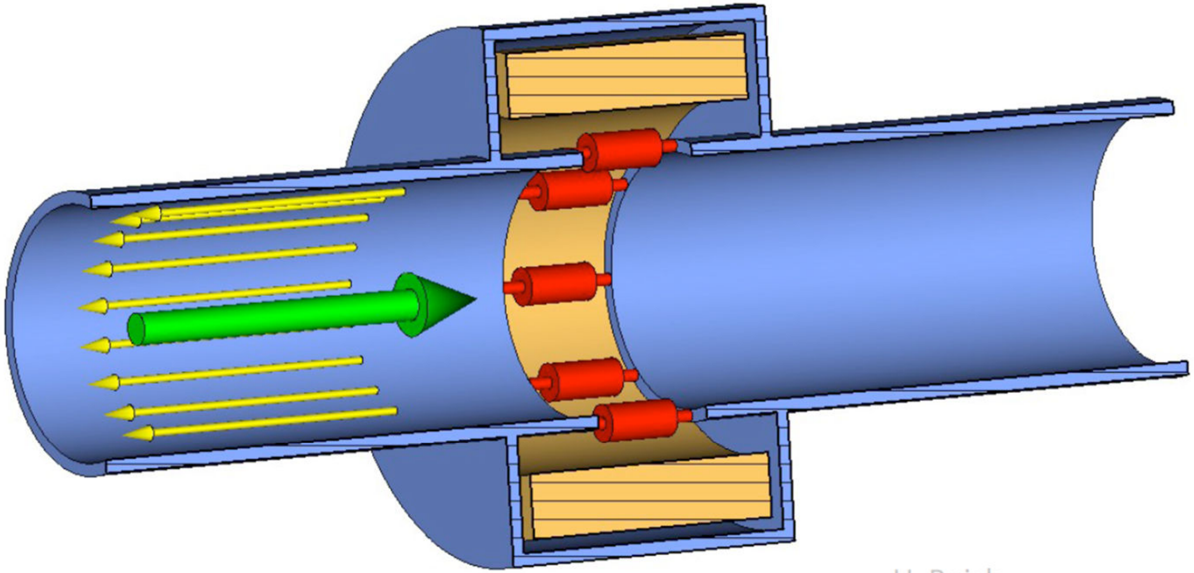
Transient driven by pickup/elec. response (MHz-GHz)





Signal amplitude still related to peak of excitation (***bunch charge***)

Wall current monitor



The **BEAM** current is accompanied by its **IMAGE** current

A voltage proportional to the beam current develops on the **RESISTORS** in the beam pipe gap

The gap must be closed by a box to avoid floating sections of the beam pipe

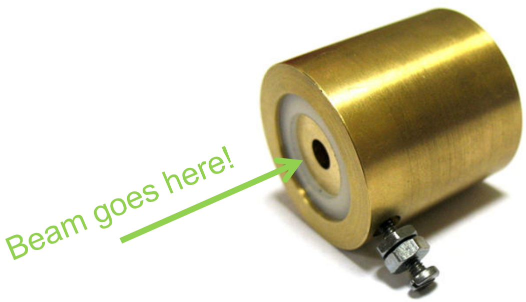
The box is filled with the **FERRITE** to force the image current to go over the resistors

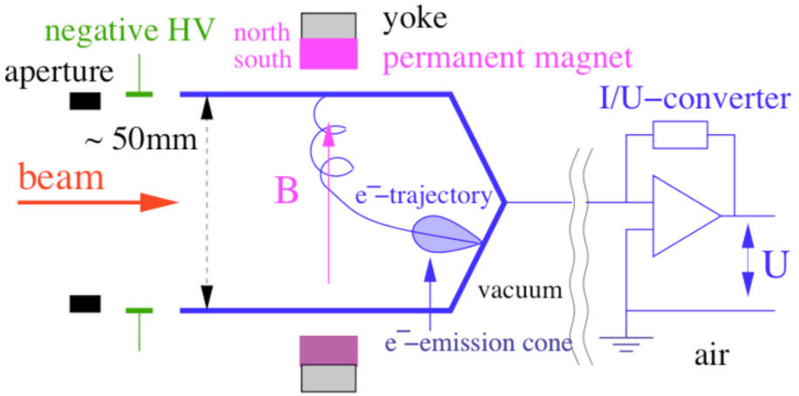
The ferrite works up to a given frequency and lower frequency components flow over the box wall

Ref [2]

Faraday Cup

Destructive method: Conductive target ionized by beam impact, measure discharge





- ~DC coupled: If only R, signal is $U = I_{beam} * R$
- High sensitivity
- Beware secondary emission: Long cup, HV suppression, or *B* field
- Must have proper termination, very high voltages (beam potential)
- Must handle full beam power (MW for high-rate beams)

Ref [2]

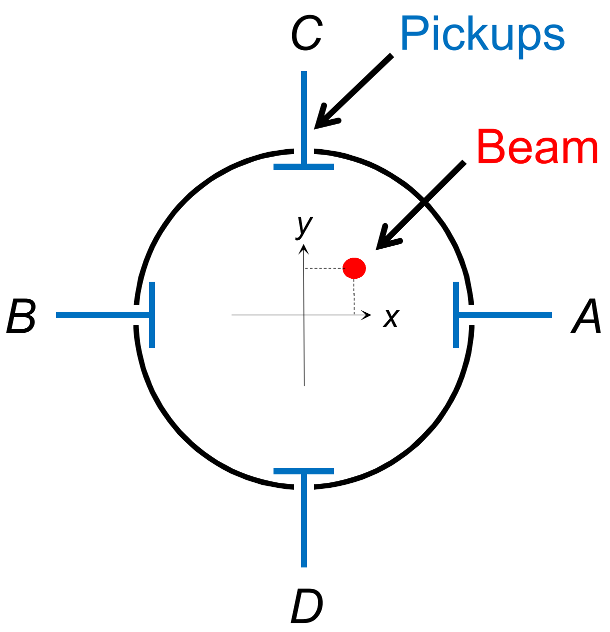
Electron Beam Position Monitors (BPMs)

Life without BPMs…

…shooting blind!

BPMs

- Four-point signal measurement
- Passing beam induces electrical transients in pickups
  - Response similar to charge monitor, engineered for pos. precision



"Difference over sum:"

$$x \propto \frac{A-B}{A+B}, \qquad y \propto \frac{C-D}{C+D}$$

Bonus: $A+B+C+D \propto \text{Charge}$

Ref [1,3]

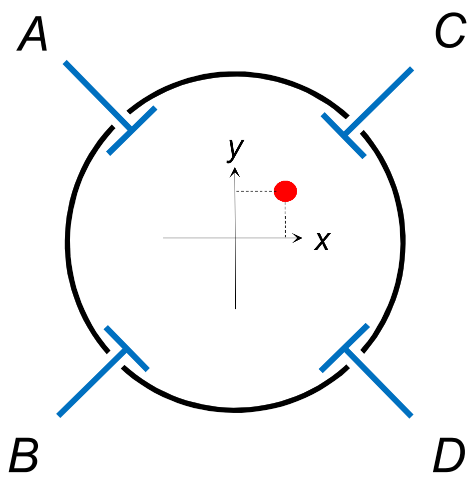
BPMs

- Synchrotrons typ. rotate 45°
  - Avoid synchrotron rad. damage

Same principle:

$$x \propto \frac{(C+D)-(A+B)}{A+B+C+D}$$

$$y \propto \frac{(A+C)-(B+D)}{A+B+C+D}$$



Ref [1,3]

Pickup type selection

Some considerations for pickup type selection:

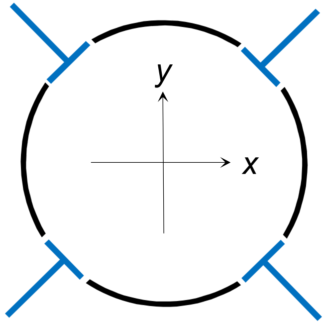
- Spatial resolution
- Beam aperture
- Electronic response
- Impedance seen by beam
- Cost (typ. *many* BPMs in any given beamline)

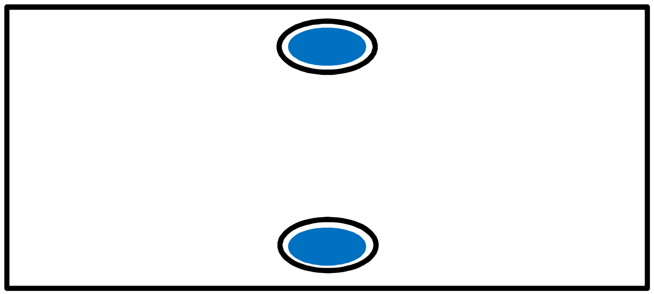
Ref [1,3]

Pickup options

1) Button type

Lower impedance, good for many turns (rings)



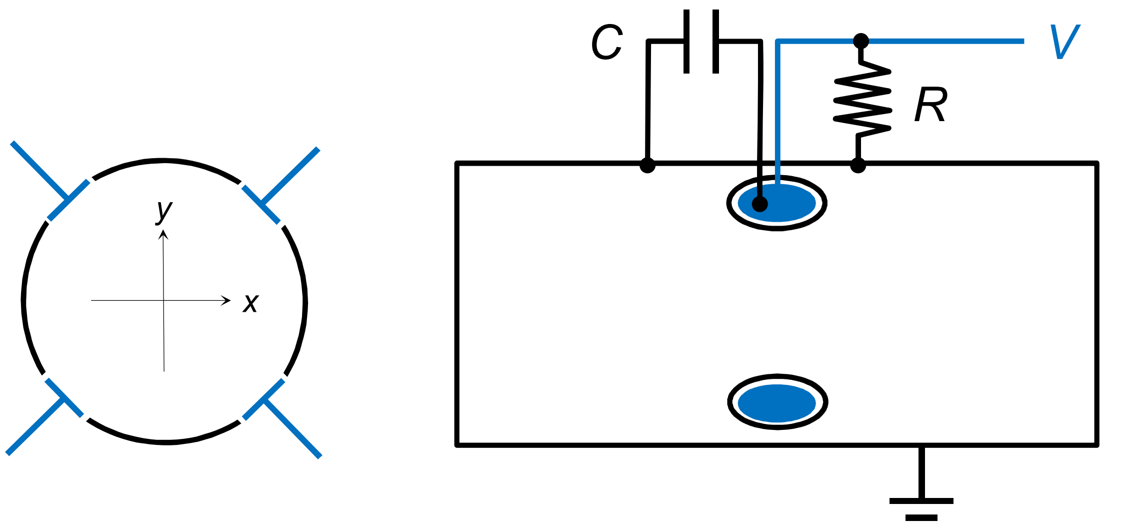


Ref [1,3]

Pickup options

1) Button type

BPM looks like high pass with characteristic frequency ω_c = 1 / RC, typically ns-scale response



Ref [1,3]

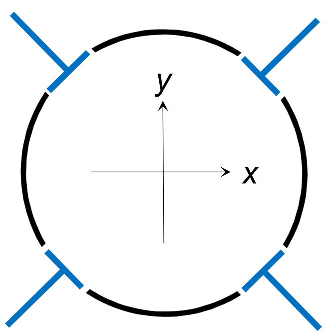
Pickup options

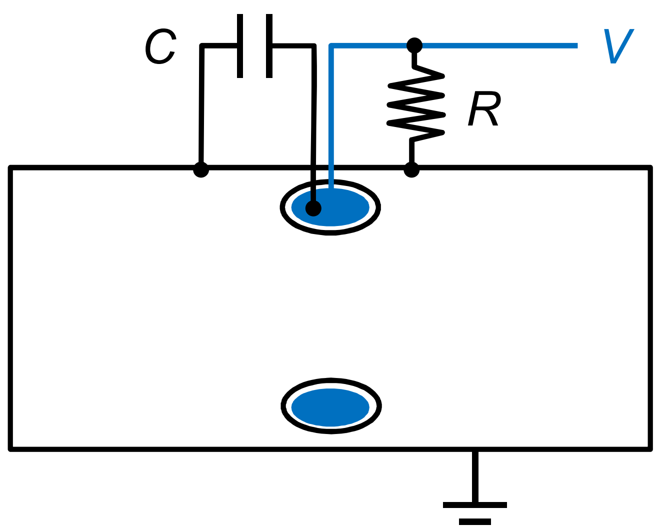
1) Button type

Typical characteristics:

50 mm aperture, ns response,

few μm resolution, moderate cost



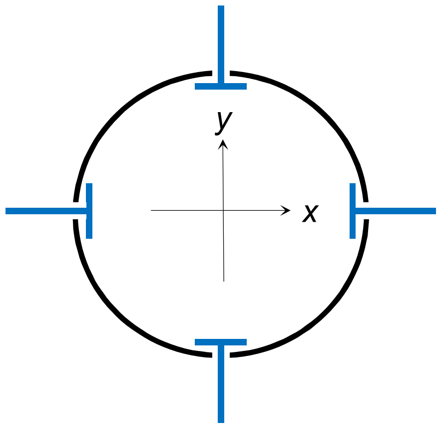


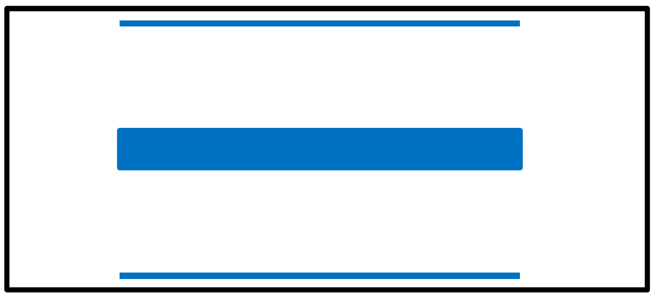
Ref [1,3]

Pickup options

2) Stripline

Higher impedence, better for single pass beams





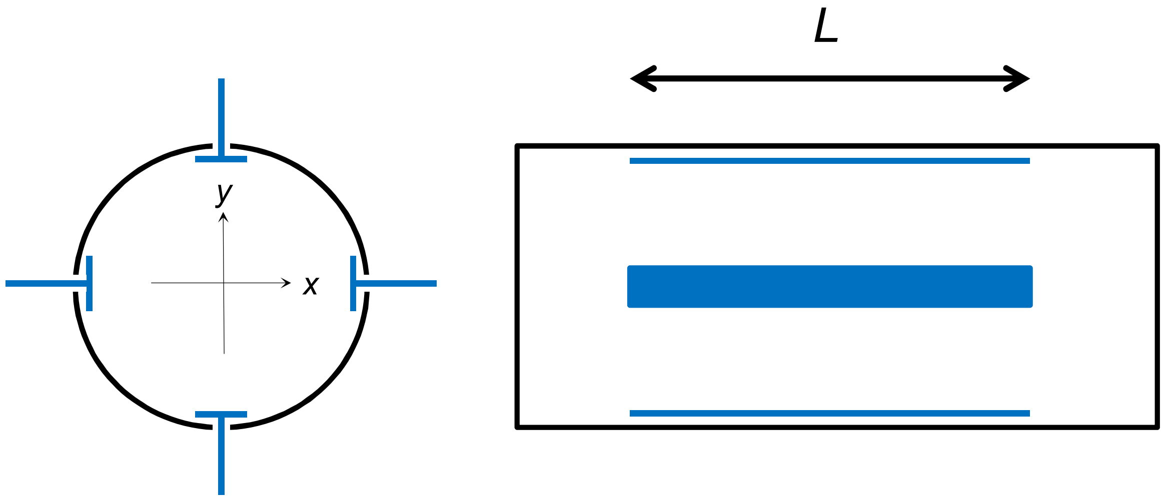
Ref [1,3]

Pickup options

2) Stripline

Characteristic time $2L / c$

Match length to bunch duration as needed



Ref [1,3]

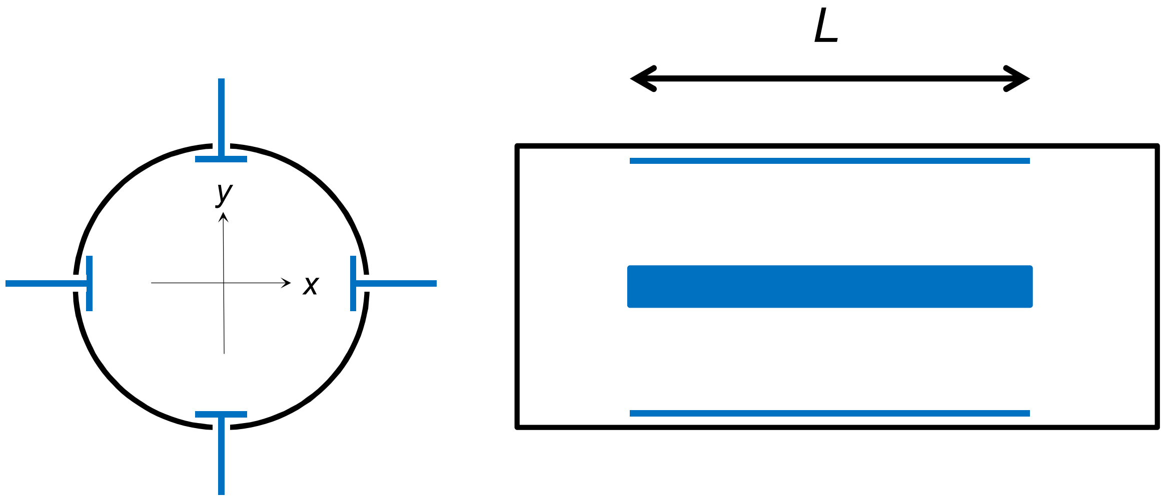
Pickup options

2) Stripline

Typical characteristics:

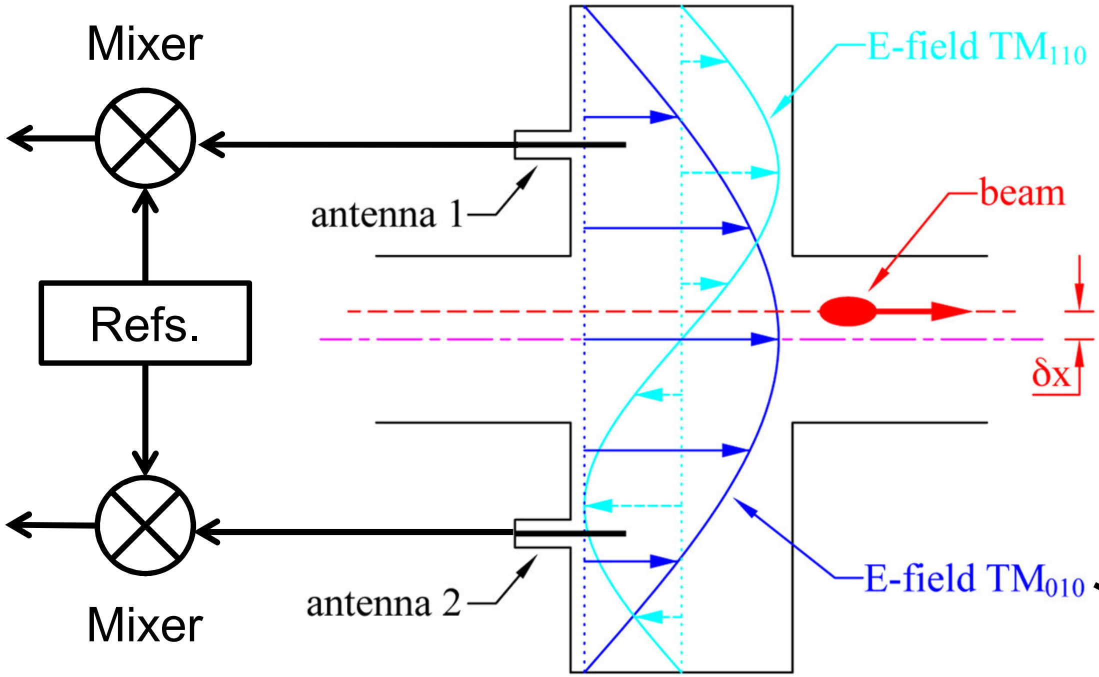
25 mm aperture, fast response,

< 1 μm resolution, medium cost

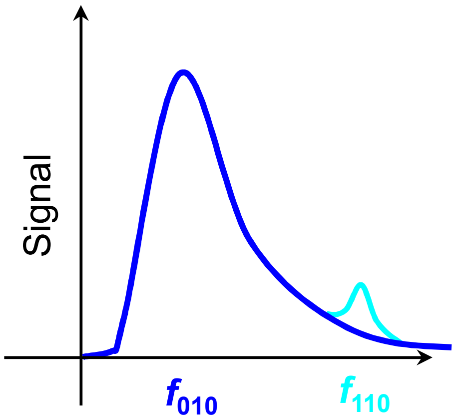


Ref [1,3]

3) Cavity



Dipole cavity modes excited by off-axis beam



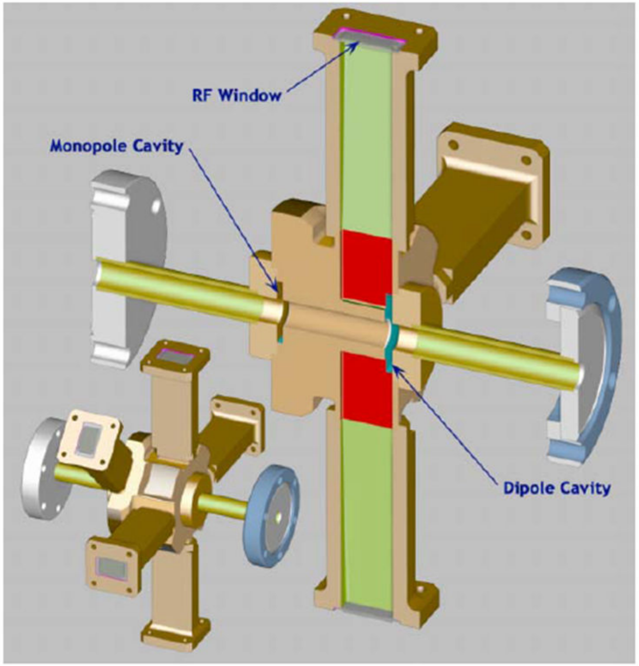
Monopole mode picked up for amplitude

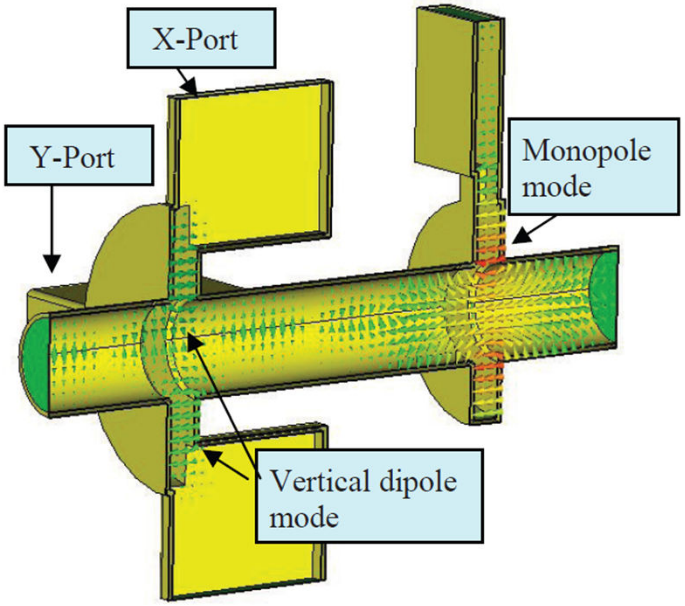
From FNAL ILC cold cavity BPM design

Ref [1,3]

Pickup options

3) Cavity





LCLS X-band cavity BPM

Excellent sensitivity, < 1 μm resolution, higher cost

Ref [1,3]

Physics with BPMs

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- Modern survey methods allow for positioning optics to within fraction of mm
- HXR FEL undulator requires positioning to within µm
- “Beam-based alignment”: Adapted from synchrotron methods, correct optics offsets [4]:

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Nuclear Instruments and Methods in Physics Research A 429 (1999) 407–413

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Section A

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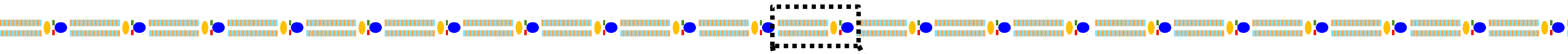
Beam-based alignment for the LCLS FEL undulator☆

P. Emma*, R. Carr, H.-D. Nuhn

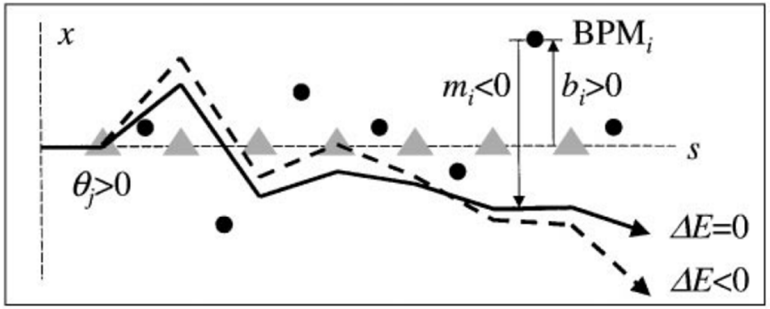
Stanford Linear Accelerator Center, Stanford, CA 94309, USA

LCLS Undulator (before self-seeding and Delta days)

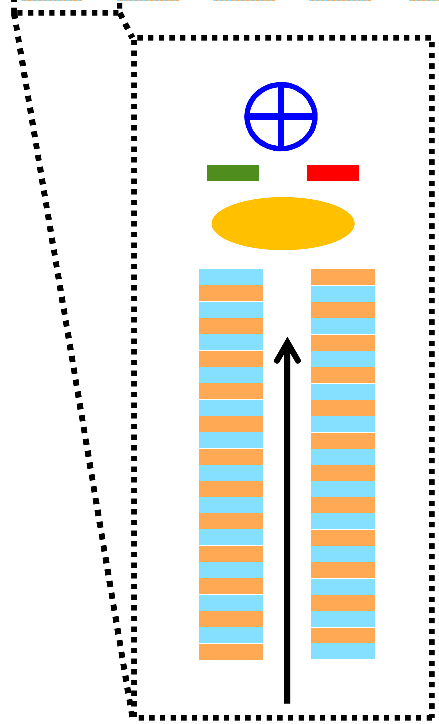
SLAC



Offset quad = *E*-dep. dipole kicks
(Und. dipole errors as well)
BPM offsets = *E*-independent



▲ Beam kicks ● BPM reading



$$m_i = \sum_{j=1}^{i} \theta_j C_{ij} - b_i$$

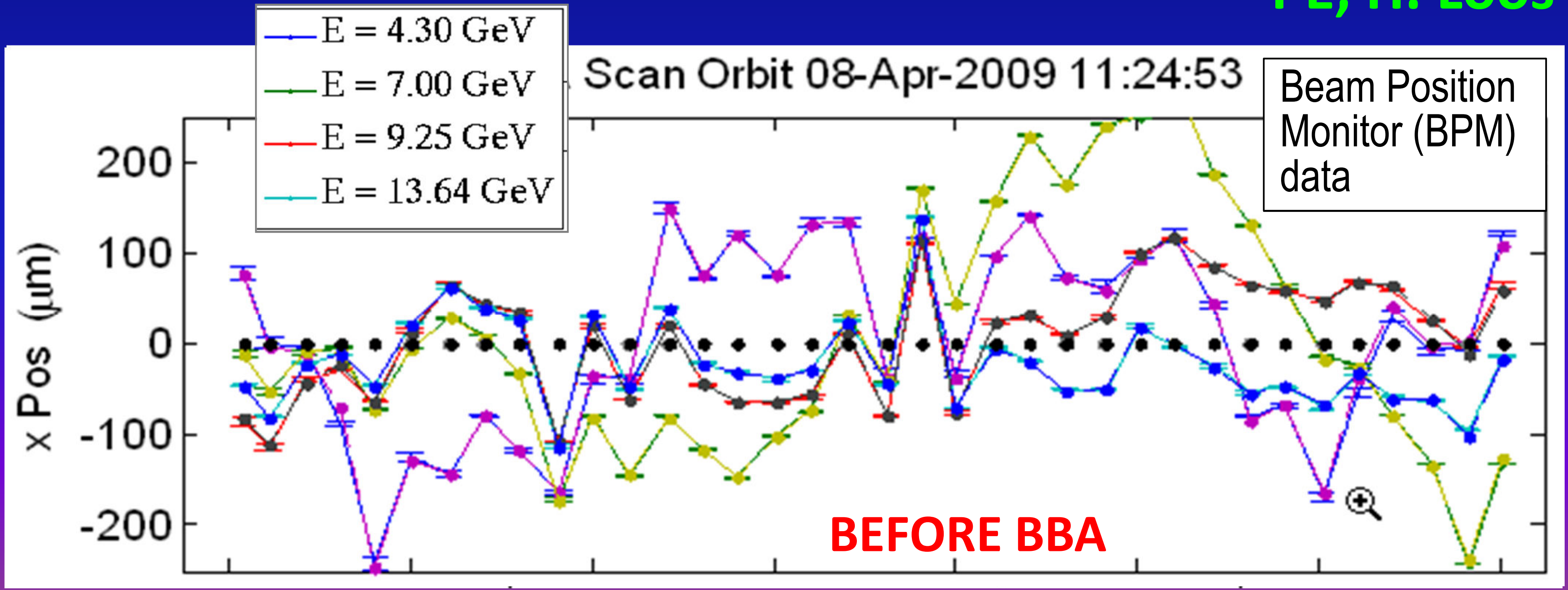
Fix quad strength, meas. BPMs, change *E*, minimize offsets and corr. by SVD

(Synchrotron: One *E*, quads varied)

Beam-Based Undulator Alignment (BBA)

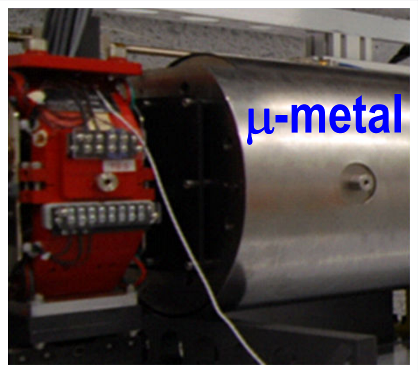
- Measure undulator trajectory at 4 energies (4, 7, 9, & 14 GeV)
- Scale all upstream (linac) magnets for each energy
- Do not change **anything** in undulator (adjust launch conditions)
- From BPM data, calculate quad and BPM alignment... (software)
- Move quads and adjust BPM offsets for dispersion free trajectory
- Iterate... (~4 hrs, once per 2-3 weeks)

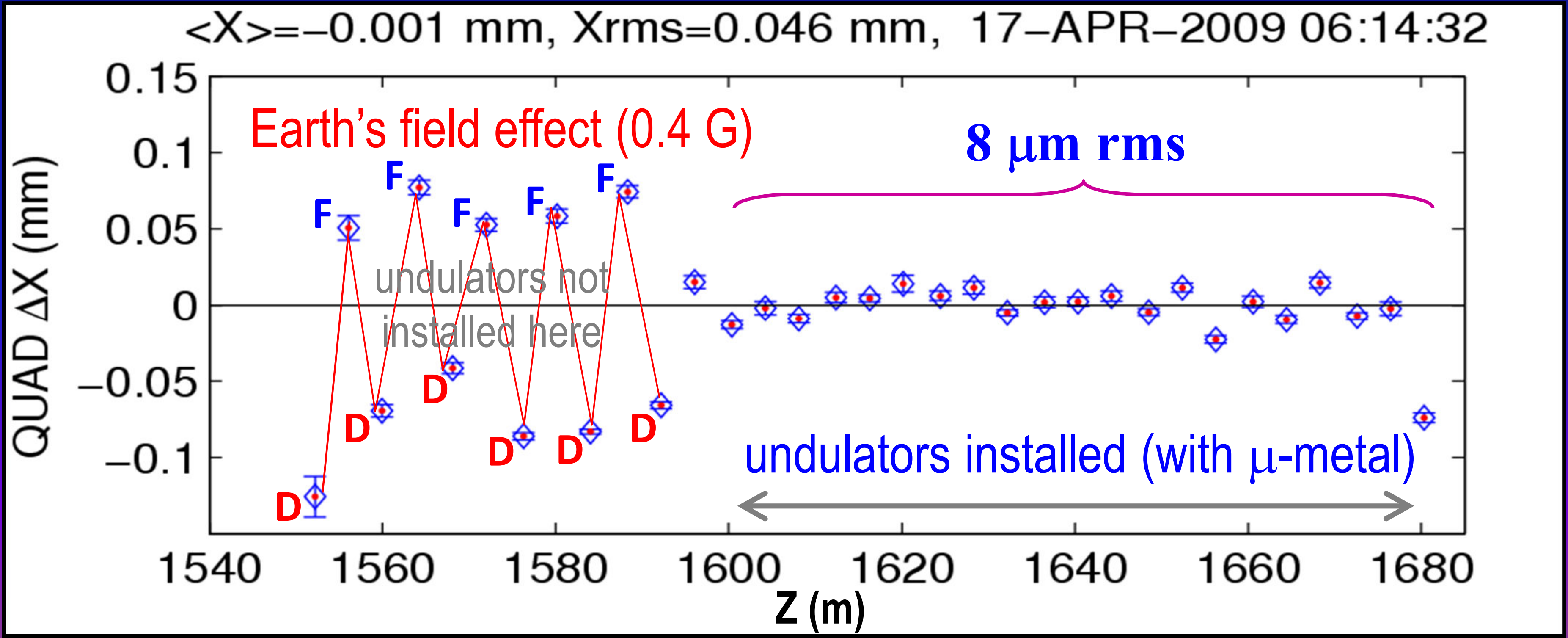
PE, H. Loos



Undulator Quadrupole Alignment after BBA

- Vary each quadrupole magnet gradient by 30% sequentially
- Record induced kick angle using BPMs
- Calculate quadrupole magnet transverse offsets (plotted below using 14-GeV data)





Electron beam diagnostics for high brightness beams

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- Beam location
  - Charge and position measurement devices

- Transverse space / shape and emittance
  - Screen monitors
  - Wire scanners

- Longitudinal *t* profile
  - Frequency domain techniques
  - Time domain techniques

Transverse phase space

2D ε : Area (action) occupied by particles in trace space

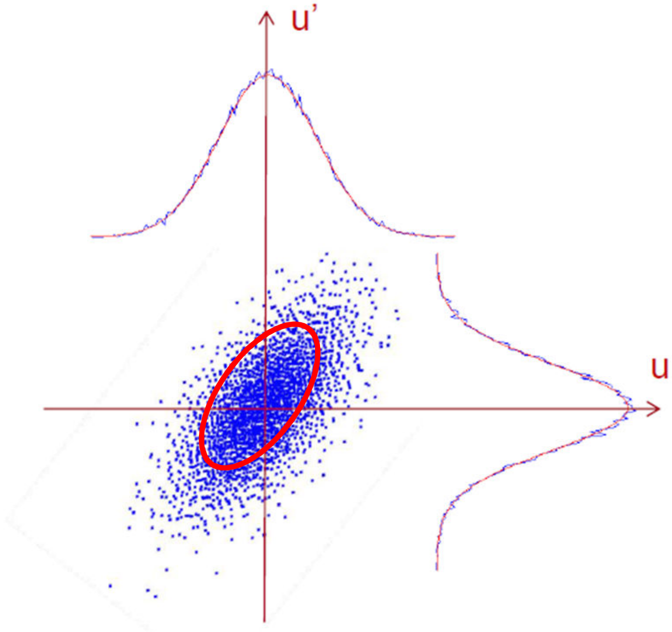
How to define for points?

- Define 1-σ contour

Usually ~Gaussian

If not, define by moments:

$$< u^2 >, < u'^2 >, < uu' >$$



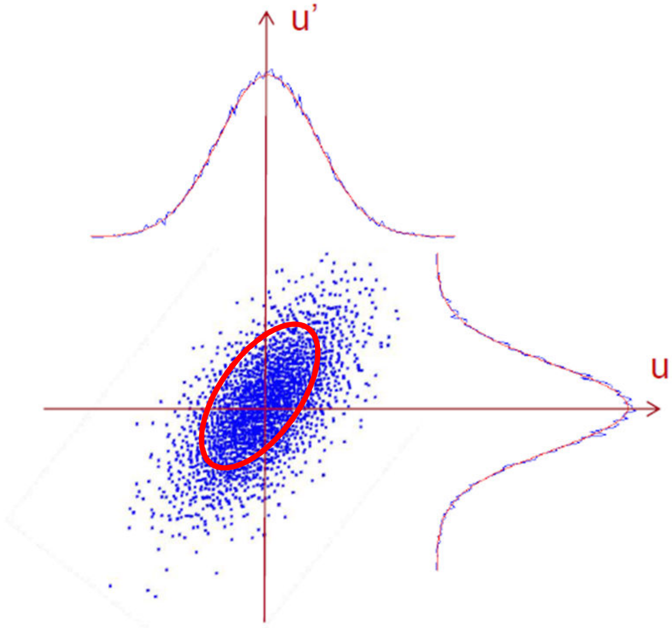
Ref [5]

Transverse phase space

Determinant of matrix of second order moments:

$$\begin{vmatrix} <u^2> & <uu'> \\ <uu'> & <u'^2> \end{vmatrix}^{1/2}$$

$$\varepsilon = \sqrt{<u^2><u'^2>-<uu'>}$$



Ref [5]

Transverse phase space

Define Twiss parameters β, α, γ by

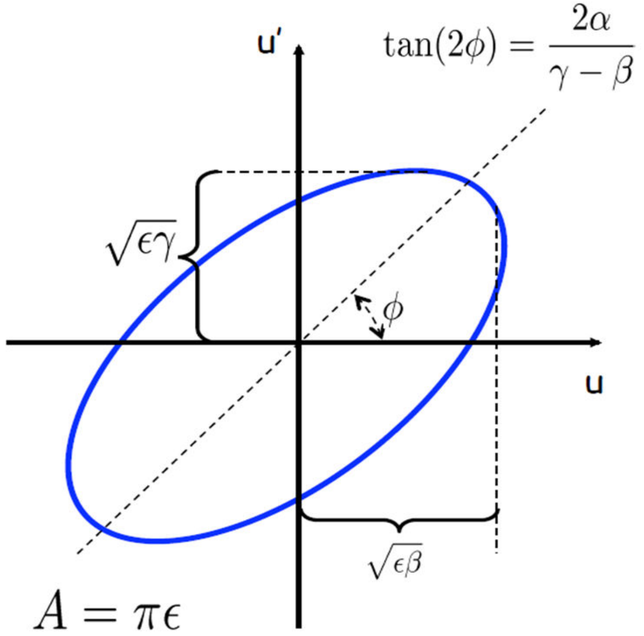
$$< u^2 > = \beta\varepsilon$$

$$< u'^2 > = \gamma\varepsilon$$

$$< uu' > = -\alpha\varepsilon$$

β, α, γ define the orientation of the ellipse

$\pi\varepsilon$ defines the area



Ref [5]

Transverse phase space

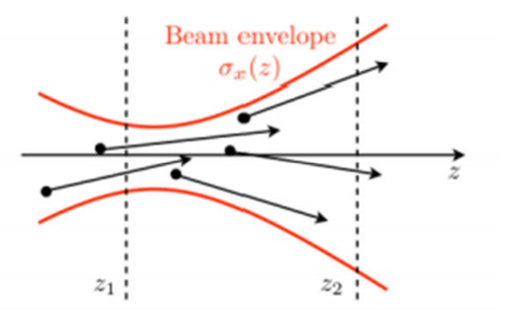
Particles (u,u') subject to restoring forces K(s) u (e.g.--periodic focusing)

$u'' + K(s)u = 0$ Hill's equation

Solution: $u(s) = \sqrt{\varepsilon\beta(s)}\, sin(\psi(s) + \phi_o)$

with $\psi(s) = \int_{s_o}^{s} \frac{ds'}{\beta(s')}$

ε and ϕ_o are constants of integration



Defining : $\alpha(s) = -\frac{1}{2}\beta'(s)$ and $\gamma(s) = \frac{1+\alpha^2(s)}{\beta^2(s)}$

$\gamma(s)u^2(s) + 2\alpha(s)u(s)u'(s) + \beta(s)u'^2(s)$ = constant (Courant-Snyder Invariant) = ε

~Analogous to Gaussian beam evolution in laser physics

Ref [5]

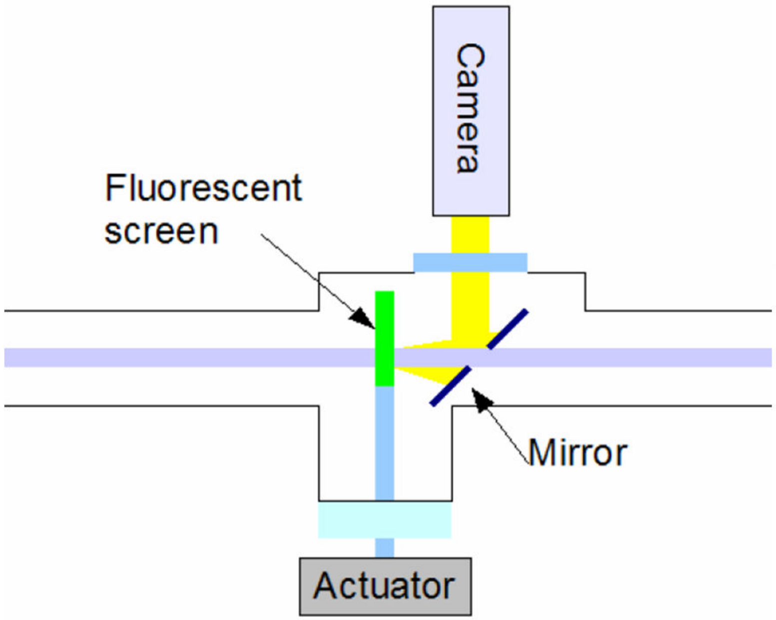
Beam profiling: Fluorescent screen

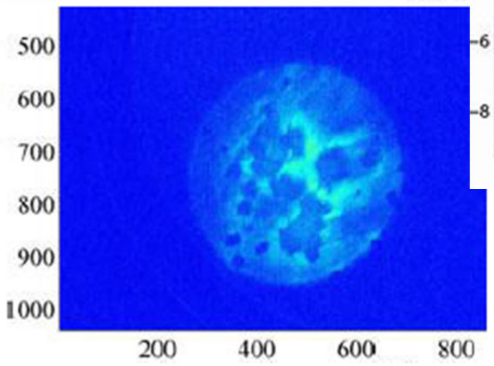
Seeing electron beam transversely…

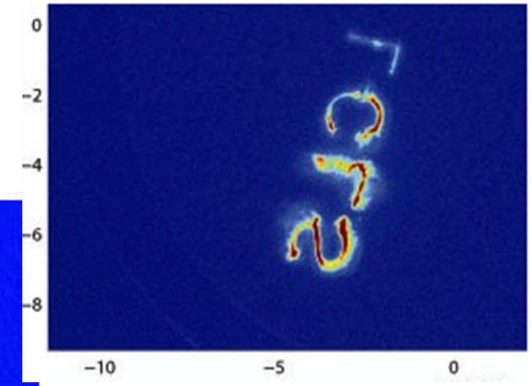
Fluorescent target = visible light where irradiated by beam

- Slow (ns) visible pulse
- Bright signal
- Wide angular distribution

Beam profiling: Fluorescent screen







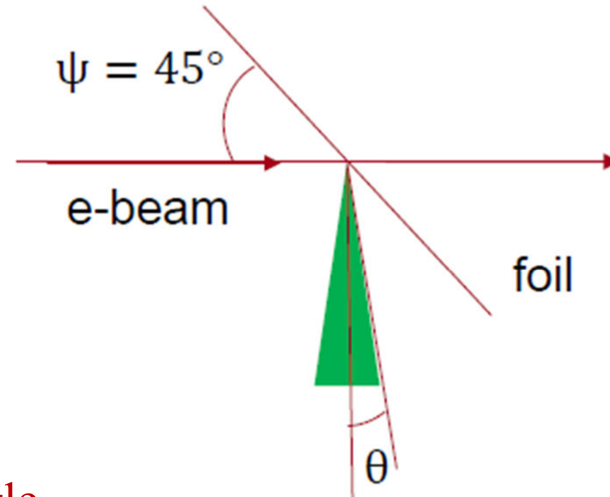
- Crystal thickness relative to beam limits resolution
- Fluorescence saturates at high densities (>0.04 pC/μm^2)
- Most useful for low-*E*, large beams (injectors)

Ref. [6]

Beam profiling: Optical transition radiation imaging

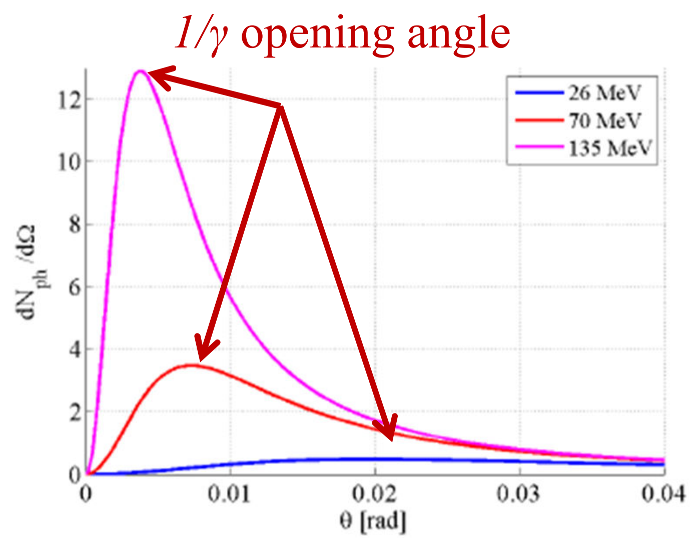
SLAC

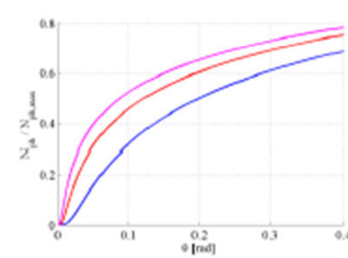
- Emitted when a relativistic electron passes boundary between materials of different electrical properties



- OTR emission from a single electron
  - Travelling with relativistic velocity $\beta \approx 1 - \frac{1}{2\gamma^2}$
  - Hitting a foil with reflectivity $R(\omega)$ foil
  - Typically at 45 degrees w.r.t to beam direction

- $$\frac{d^2N}{d\omega d\Omega} = \frac{\alpha}{4\pi^2}\frac{R(\omega)}{\omega}\frac{\beta^2 \sin^2(\theta)}{(1-\beta\cos(\theta))^2}$$





$\theta > 150\ mrad$, $60\%\ of\ light$

Ref [5]

Beam profiling: OTR imaging

Incoherent imaging (long bunch)

Image = *PSF* * $\rho(x,y)$

Intensity prop. to N_e

$$|\boldsymbol{E}_{S,N}(\boldsymbol{r})|^2 = N \int d^2 r' \, dz \;\; \rho(\boldsymbol{r}', z) |\boldsymbol{E}_{S,N}(\boldsymbol{r} - \boldsymbol{r}')|^2$$

$$+ N^2 \left| \int d^2 r' dz \;\; e^{-ikz} \rho(\boldsymbol{r}', z) \boldsymbol{E}_{S,N}(\boldsymbol{r} - \boldsymbol{r}') \right|^2$$

Coherent imaging (short bunch, comp. to λ)

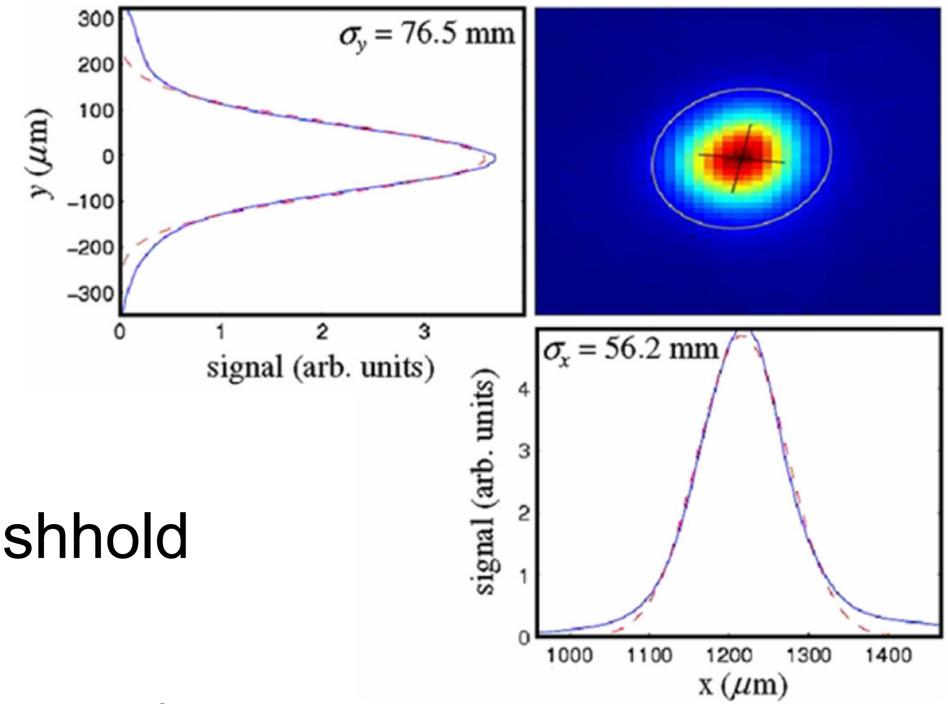
Image formation linear *in complex field*

In short $FT^{-1}\left(E_s(k_x, k_y) \times \rho(k_x, k_y)\right)$

Intensity prop to N_e^2 much brighter than incoh.

Ref [5]

Beam profiling: Incoherent OTR



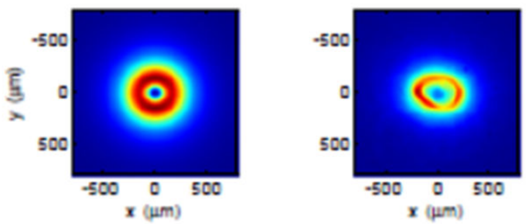
- Works to foil damage threshhold (dense beam)
- Emission prompt (bunch length)
- Res. limit by optics & 1/γ angle
- Gets distorted if coherent…

Ref. [6]

Beam profiling: Coherent OTR

Coherent emission:

- OTR image show square of gradient of shape
  - thus the “doughnut” shape



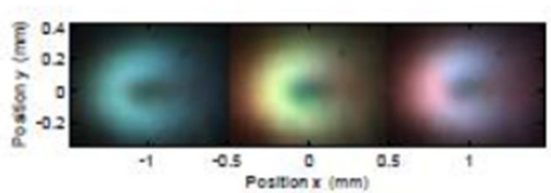


Figure 10: Series of COTR color images taken within one minute showing varying color content from shot to shot.

Visible-light coherence from short bunches and/or microbunching instability

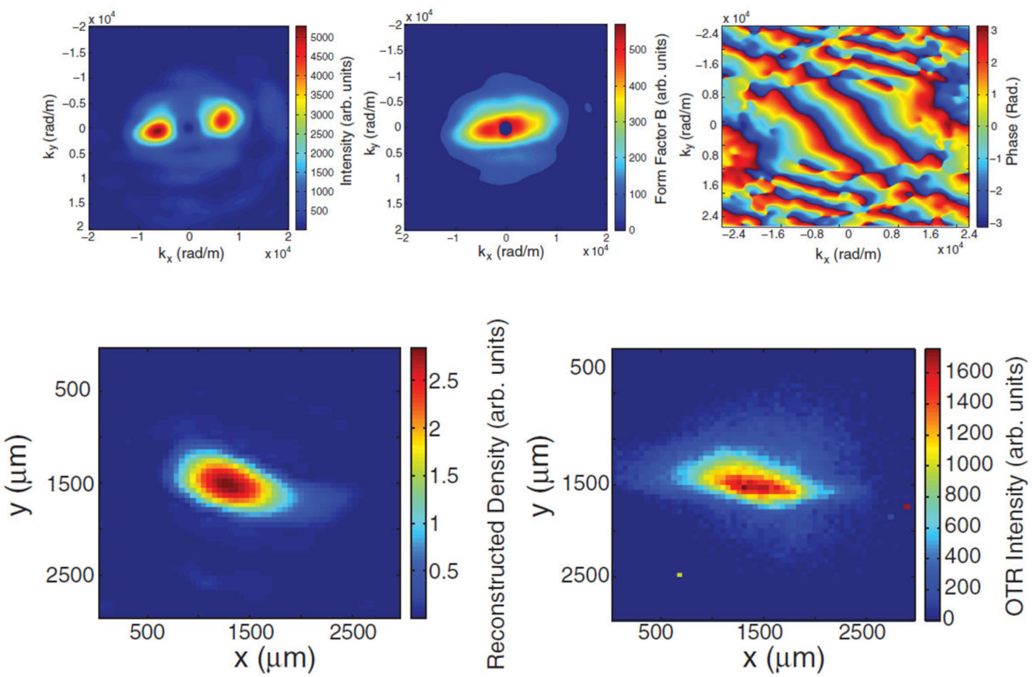
Spoils measurement of rms beam size (under estimate)

Ref [5] [7]

COTR mitigation schemes

Spectral separation:	Shorter wavelengths (EUV)
Angular separation:	Scintillator tilted to avoid directional COTR
Temporal separation:	Scintillator screen (that slow, ns response) + fast-gated ICCD camera

Far-field transverse phase reconstruction [9]



Ref [5,8]

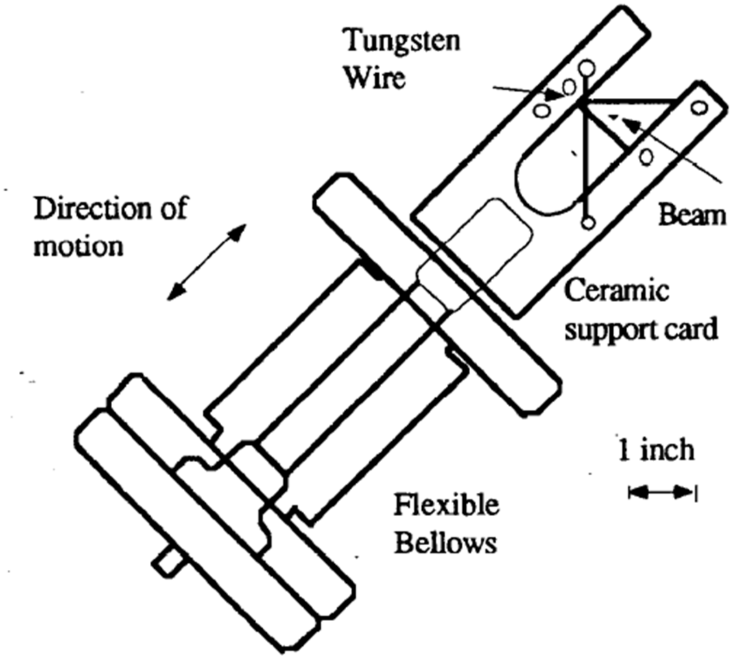
Beam profiling: Wire scanner

Wires scan through beam

x, y, and 45º profiles in one motion

Measure either:

1. Charge deposited on wires
2. Scattered electrons downstream (scintillator + PMT) → LCLS

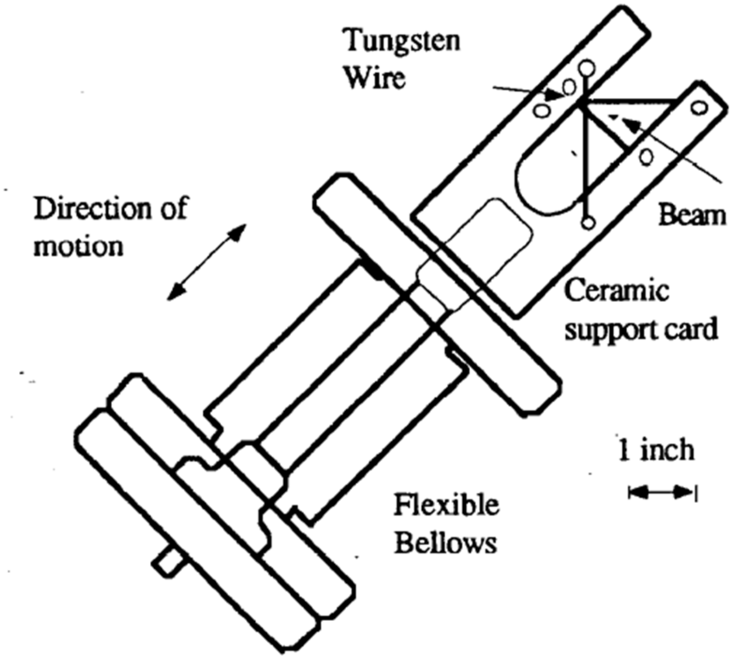


Beam profiling: Wire scanner

Characteristics:

- µm resolution (wire thickness)
- Minimally invasive
- Can handle higher power beam
- No imaging artifacts / COTR

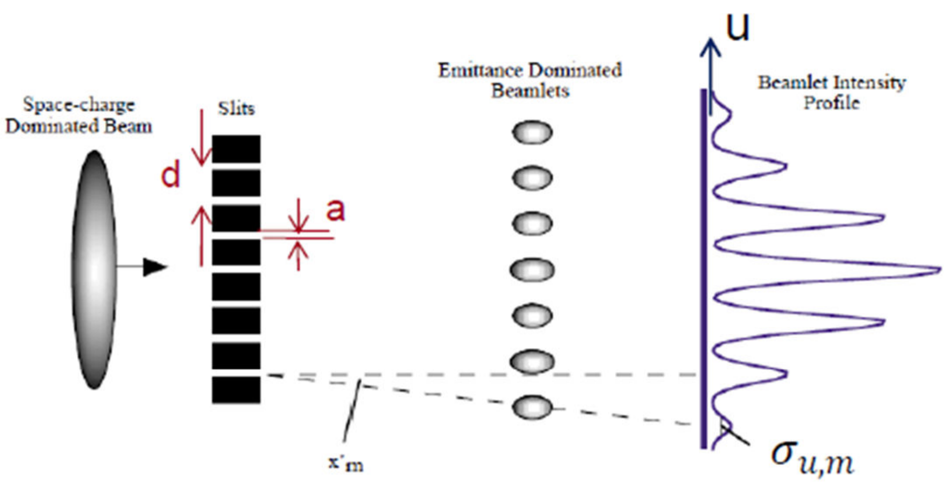
- Multi-shot, slow
- No <*xy*> (all projections)



Measuring emittance: Pepper Pot

SLAC

Multislit in high Z material



Direct reading of trace space

$$u'_m = x'_m = \frac{u_m - x_m}{L} = \frac{u_m - md}{L}$$

$$\sigma'^2_m = \sigma^2_{u,m} - \left(\frac{a}{4}\right)^2$$

$$< x^2 > = \frac{\sum I_m x_m^2}{\sum I_m} ,$$

$$< x'^2 > = \frac{\sum I_m \left(x'^2_{m,o} + \frac{\sigma'^2_m}{L^2} \right)}{\sum I_m} ,$$

$$< xx' > = \frac{\sum I_m x_{m,o} x'_{m,o}}{\sum I_m}$$

With I_m detected intensity for m^{th} beamlet

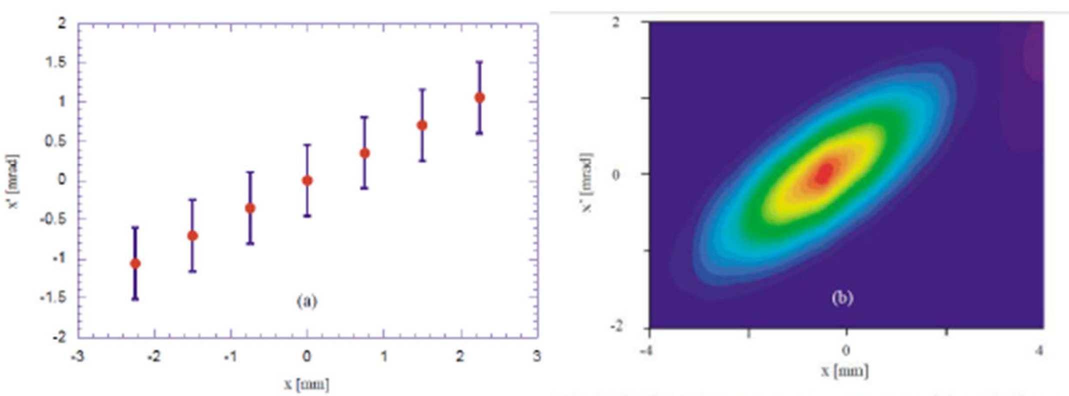


FIG. 2. (Color) (a) Beam trace space constructed from the beamlet intensity profile illustrated in Fig. 1. Each point represents the position of a beamlet in trace space and the error bars indicate the thermal spread of the beamlets. (b) Contour plot representation of the same data. Here the relative weights of the beamlets are resolved.

M. Zhang, Report No. FERMILAB-TM-1988, 1996
S. G. Anderson, et al. Phys. Rev. ST Accel. Beams 5, 014201 (2002)

Ref [5]

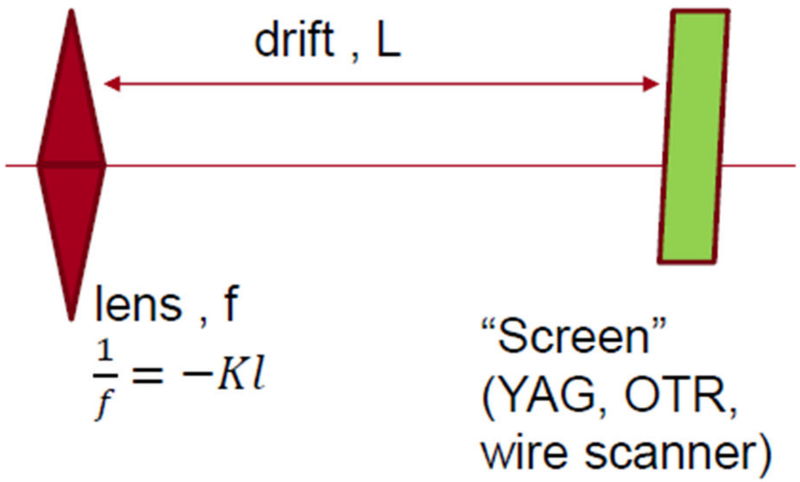
Measuring emittance: Quad Scan

Quadrupole/solenoid scan

$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix} \begin{pmatrix} x_o \\ x'_o \end{pmatrix} = R \begin{pmatrix} x_o \\ x'_o \end{pmatrix}$$

$$\begin{cases} x = \left(1 - \frac{L}{f}\right) x_o + L x_o \\ x' = -\frac{x_o}{f} + x'_o \end{cases}$$

$$\langle x^2 \rangle = \left(1 - \frac{2L}{f} + \frac{L^2}{f^2}\right) \langle x_o^2 \rangle + 2L \left(1 - \frac{L}{f}\right) \langle x_o x'_o \rangle + L^2 \langle x'^2_o \rangle$$



$$\langle x^2 \rangle = \frac{a}{f^2} + \frac{b}{f} + c$$

Change optics, measure in one place

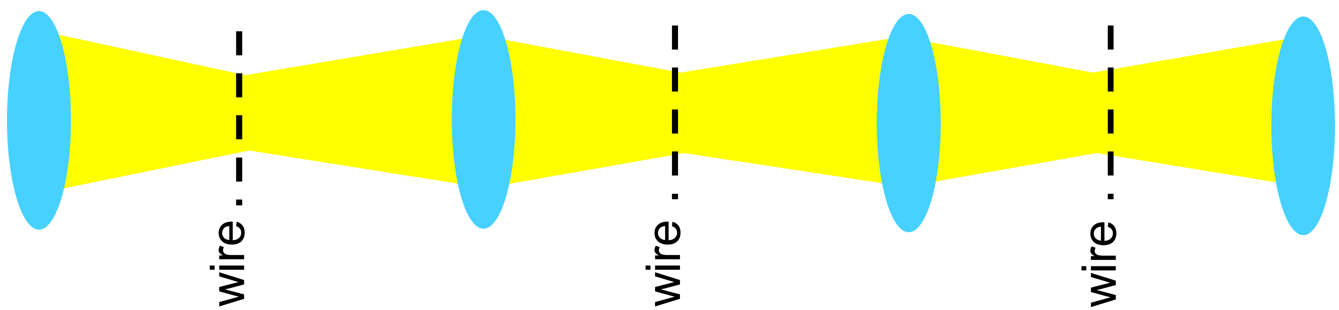
Solve for a,b,c to determine $\langle x_o^2 \rangle, \langle x_o x'_o \rangle, \langle x'^2_o \rangle$

$$\varepsilon = \sqrt{\langle x_o^2 \rangle \langle x'^2_o \rangle - \langle x_o x'_o \rangle}\ , \ \alpha = -\frac{\langle x_o x'_o \rangle}{\varepsilon}\ , \ \beta = \frac{\langle x_o^2 \rangle}{\varepsilon}\ , \ \gamma = \frac{\langle x'^2_o \rangle}{\varepsilon}$$

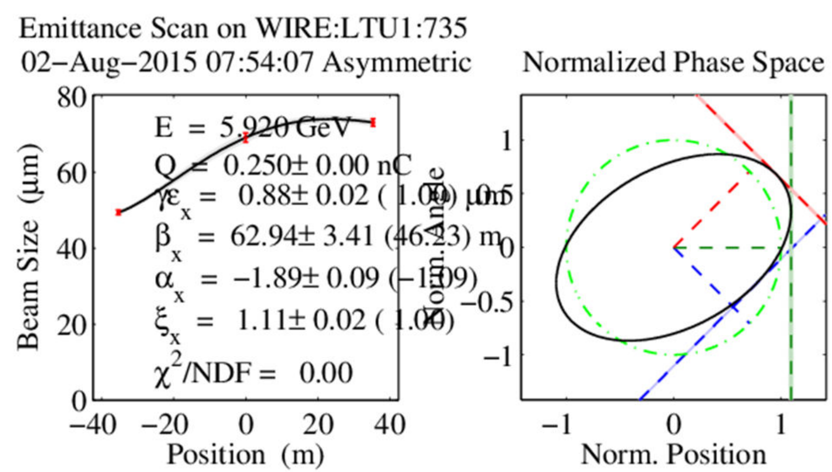
Measuring emittance: Multi-profile

SLAC

…or static optics, measure in many places



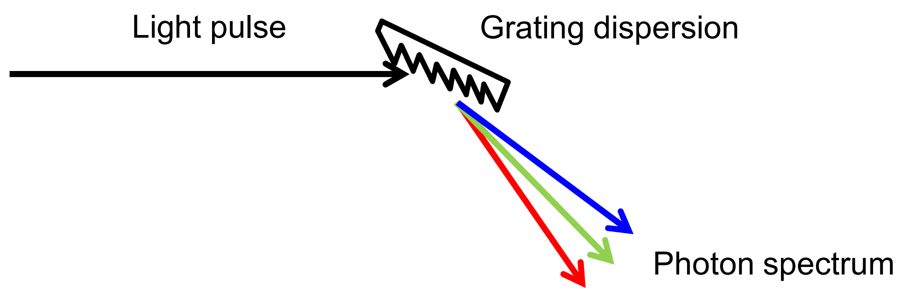
With knowledge of phase advances, reconstruct ellipse



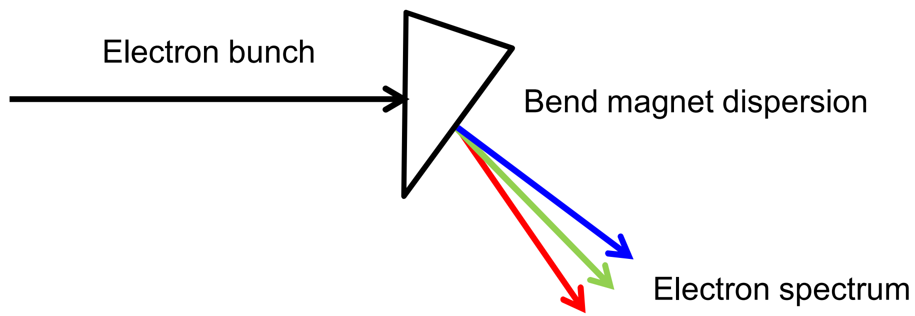
Extension to Energy Profiles

E profiles: **dispersion**

Photons:



Electrons:



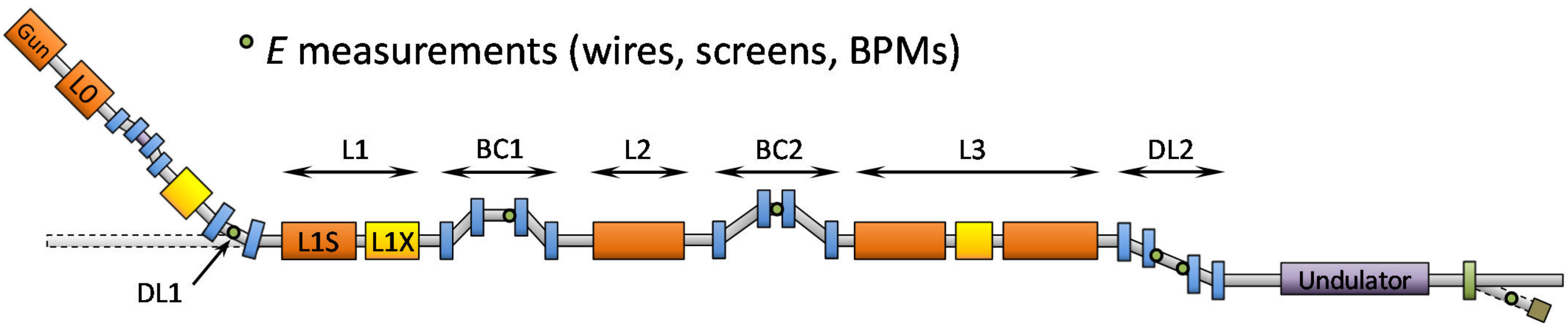
$$\Delta x / \Delta \delta = \rho$$

$$\frac{1}{\rho[\mathrm{m}]} = 0.3 \frac{B[\mathrm{T}]}{cp[\mathrm{GeV}]}$$

Extension to Energy Profiles

With dispersion ρ in x/y (min. $\beta_{x/y}$ for profiles) $E = x / \rho$

- ρ + BPM = Mean E
- ρ + Wire scanner / Screen = E spectrum



Electron beam diagnostics for high brightness beams

- Beam location
  - Charge and position measurement devices
- Transverse space / shape and emittance
  - Screen monitors
  - Wire scanners
- Longitudinal *t* profile
  - Frequency domain techniques
  - Time domain techniques

Longitudinal *t* profile

Frequency domain techniques

- Measurements of coherent beam radiation spectra
- Interferometry / spectroscopy

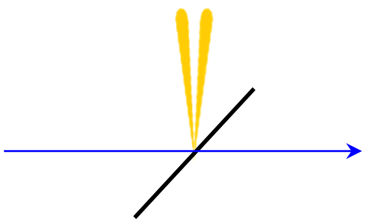
Time domain techniques

- Electro-optic sampling
- Streak camera
- Transverse deflecting mode cavities

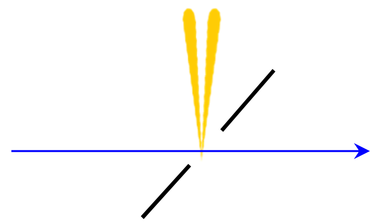
Coherent Beam Radiation

Radiative source:

Most frequency-domain techniques measure coherent beam radiation spectra



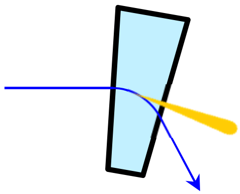
Transition (CTR)



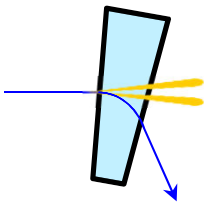
Diffraction (CDR)

CTR destructive, well-defined source

CDR less/non-interceptive, loses some time resolution



Synchrotron (CSR)



Edge (CER)

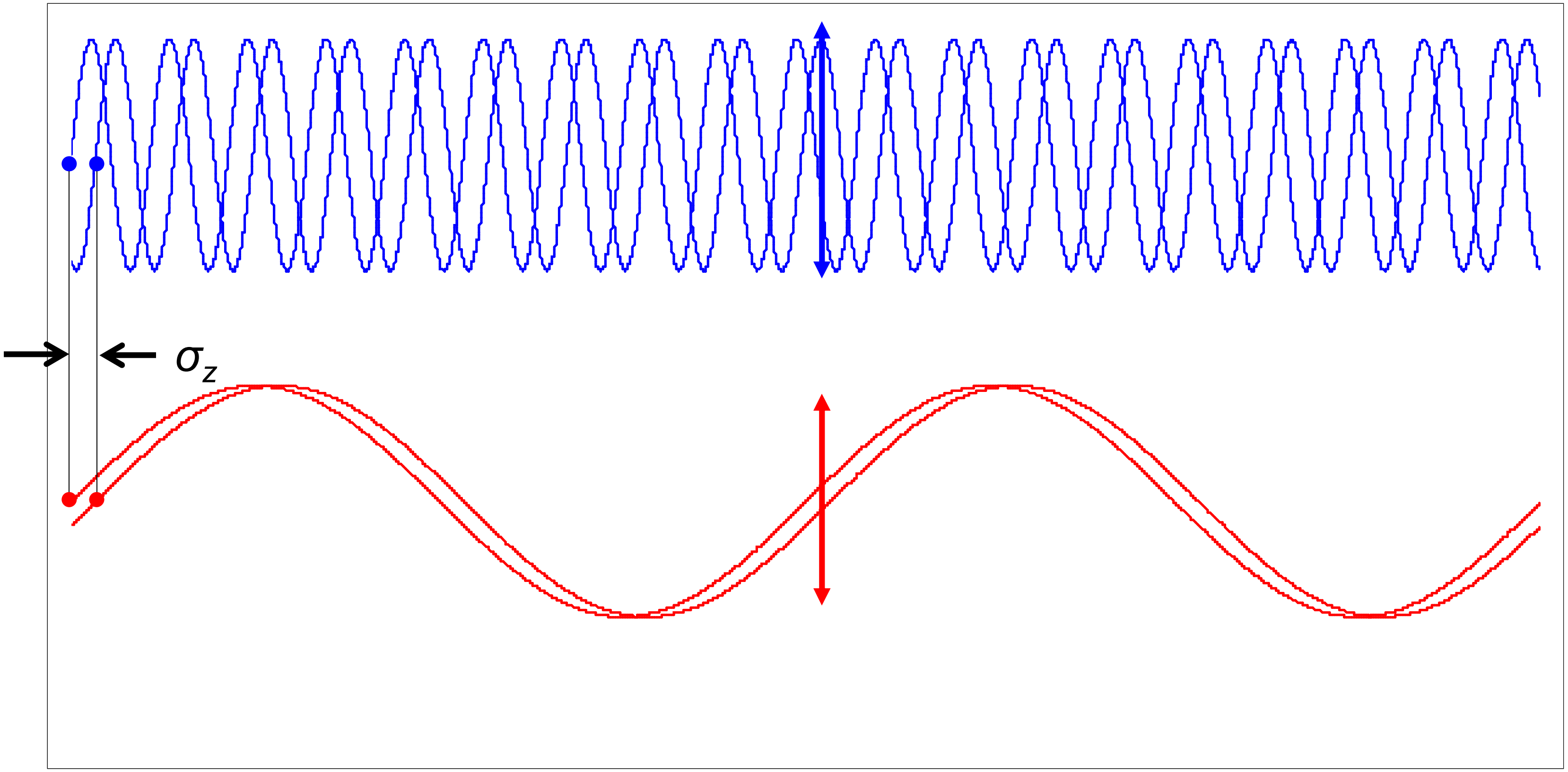
CSR/CER non-destructive, "free" at compressor where we're most interested

Comparatively ill-defined source points

Coherent spectra

SLAC

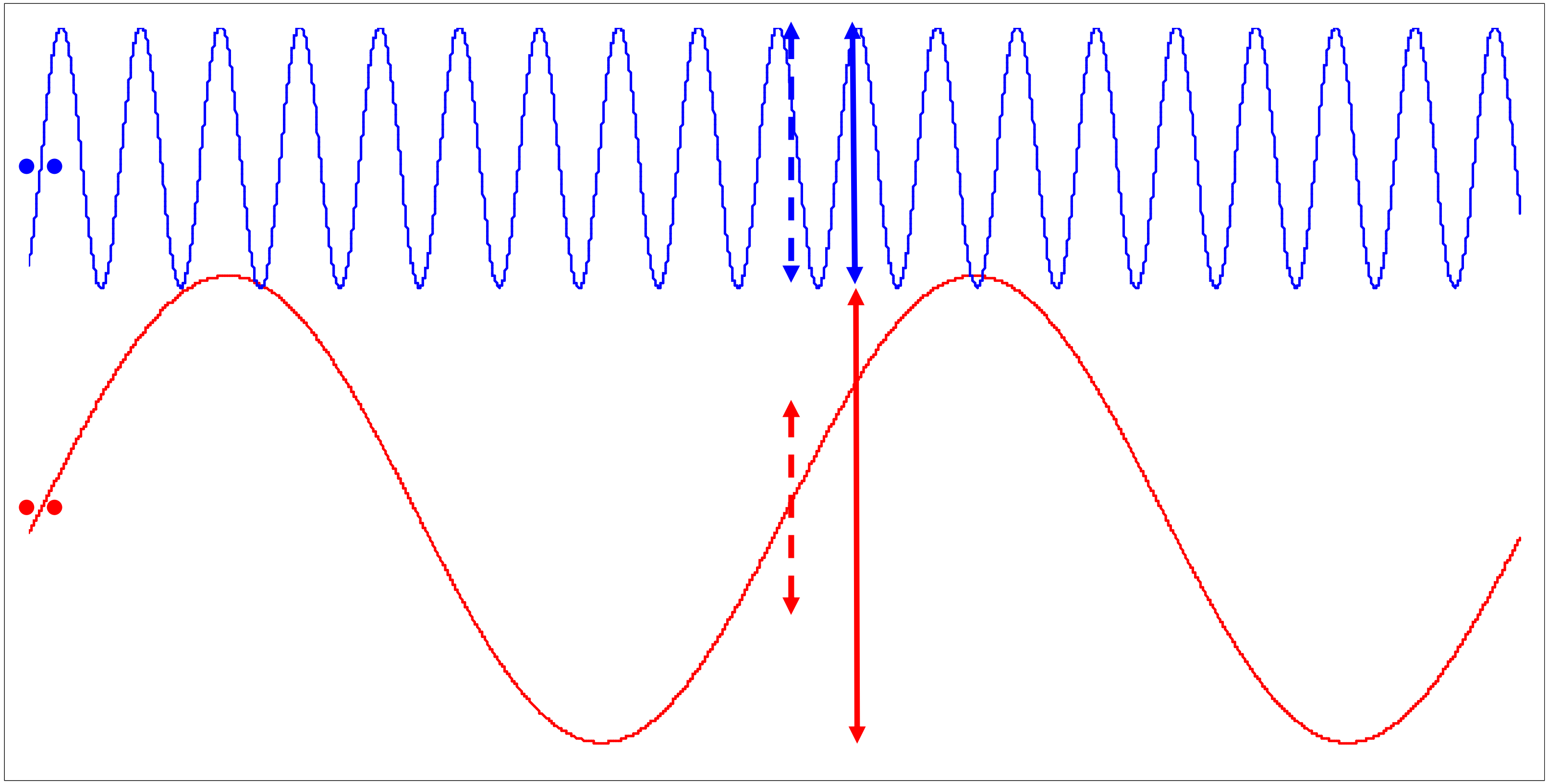
High-frequency radiation from head/tail is out of phase



Low-*f* ($\lambda \geq \sigma_z$) are in phase

Coherent spectra

High-*f* sum incoherent, intensity scales as N_e



Low-*f* sum coherently, intensity scales as N_e^2

Coherent spectra

Sum over continuous bunch profile $\rho(z)$

Spectral intensity u related to by F.T. (1D approximation):

$$u(k,\vec{r}) \propto N^2 \left|\vec{E}_e(k,\vec{r})\right|^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i[k\cdot(z''-z')]}\rho(z')\rho(z'')dz'\,dz''$$

$$= N^2 \left|\vec{E}_e(\vec{r},k)\right|^2 \left|f(k)\right|^2$$

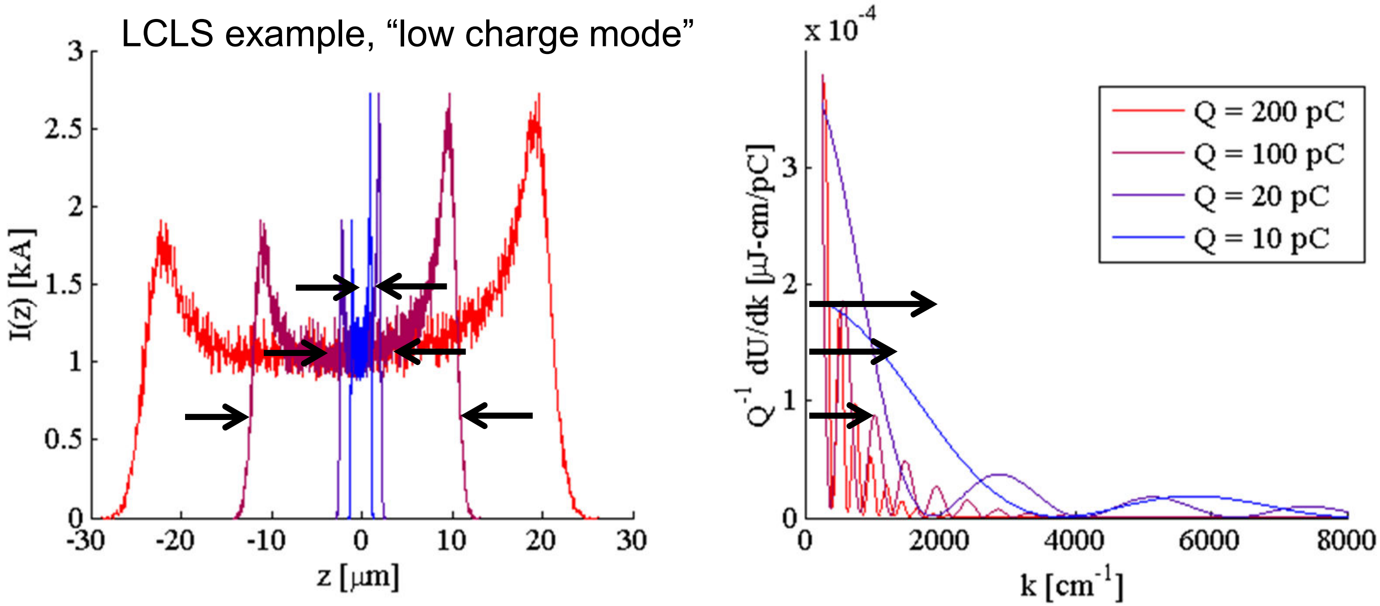
"Form factor," F.T. of current

Coherent enhancement

Single e^- imaging response

Coherent spectra

FT relation: As bunch gets shorter, spectrum will get wider
→ Bunch length-related signal



$$u(k,\vec{r}) = N^2 \left|\vec{E}_e(\vec{r},k)\right|^2 \left|f(k)\right|^2$$ ←

Caveat:

- Not directly related to *t*
- Amplitude only, no spectral phase
- Can’t invert FT to get exact *z* profile

The “detector response”

Diffraction response from $\left|\vec{E}_e(\vec{r},k)\right|^2$

- For 1s-100s fs, interesting λ = μm – mm
- Diffraction losses are high for the long wave components
- Finite apertures = “high-pass filter”

Diffraction response

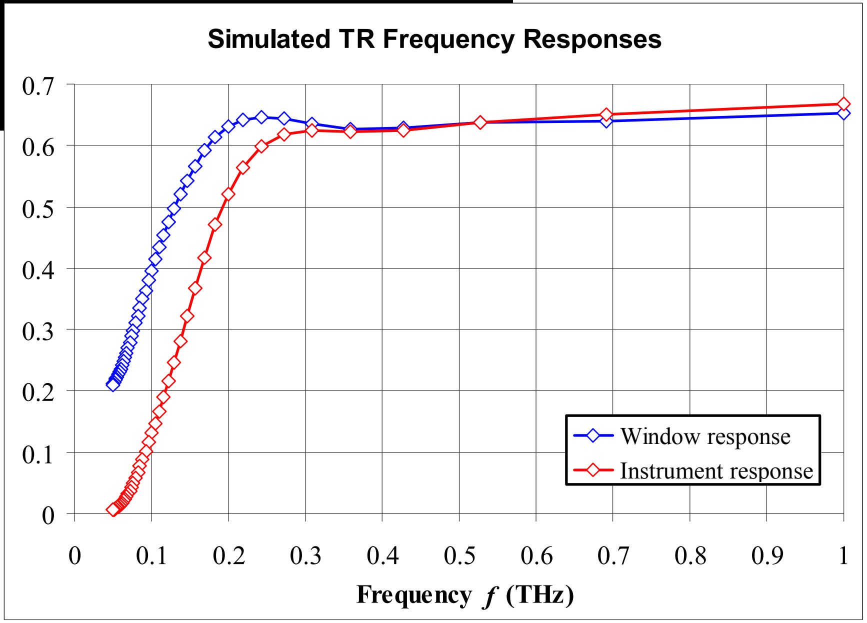
- Weizsacker-Williams "virtual quanta" source [10]:
  - Electrons illuminate system (edge/foil) with self-field
  - Fourier-transform relativistic electron field :

$$
\begin{cases}
\widetilde{E}_r(r,z_0,\omega) = \dfrac{q\omega}{(2\pi)^{3/2}\varepsilon_0\beta^2c^2\gamma}K_1\left(\dfrac{\omega}{\beta c\gamma}r\right)\exp\left(\dfrac{i\omega}{\beta c}z_0\right) \\
\widetilde{E}_z(r,z_0,\omega) = \dfrac{-iq\omega}{(2\pi)^{3/2}\varepsilon_0\beta^2c^2\gamma^2}K_0\left(\dfrac{\omega}{\beta c\gamma}r\right)\exp\left(\dfrac{i\omega}{\beta c}z_0\right) \\
\widetilde{B}_\phi(r,z_0,\omega) = \dfrac{q\omega}{(2\pi)^{3/2}\varepsilon_0\beta c^3\gamma}K_1\left(\dfrac{\omega}{\beta c\gamma}r\right)\exp\left(\dfrac{i\omega}{\beta c}z_0\right)
\end{cases}
$$

  - These are source, propagate using diffraction integrals
    - Fresnel, Kirchoff, Vector, Gauss-Hermite...
    - E.g.– Ref [11]

Diffraction response

Vector diffraction propagation of CTR @ λ = 1 mm, 250 MeV beam

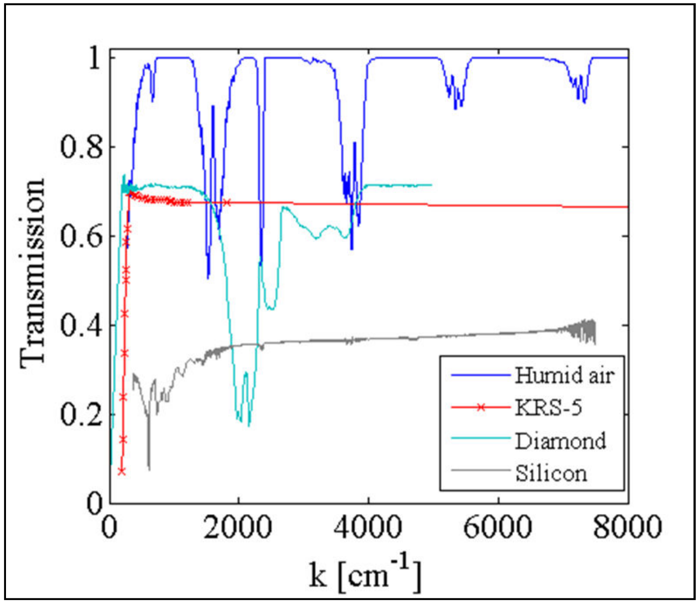


Truncation at limiting apertures yields DC losses

Detection response

Measurement usually spans at least a decade

- Optics all designed for wide wavelength range
- Frequent reliance on reflective optics
- Common transmissive materials:
  - Diamond (vacuum window), PCX, polyethylene, silicon, germanium
- H20, CO and CO2 also have IR-THz absorptions
  - Dry or vacuum purged optics

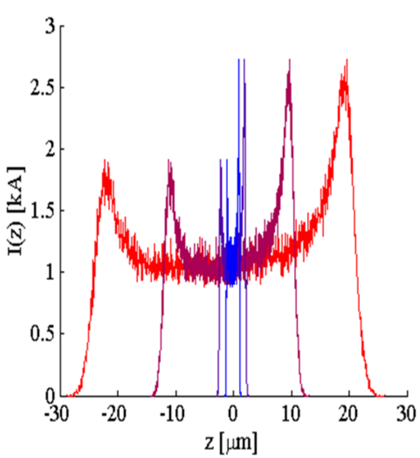


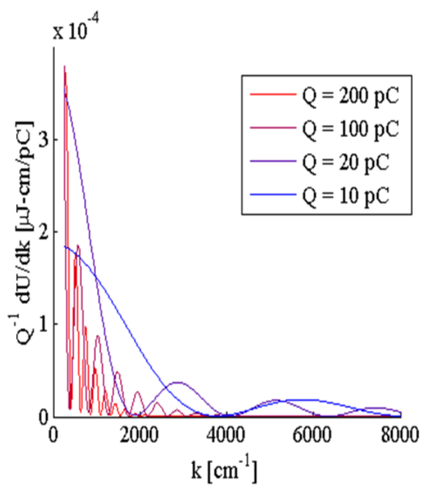
LCLS Relative Bunch Length Monitor

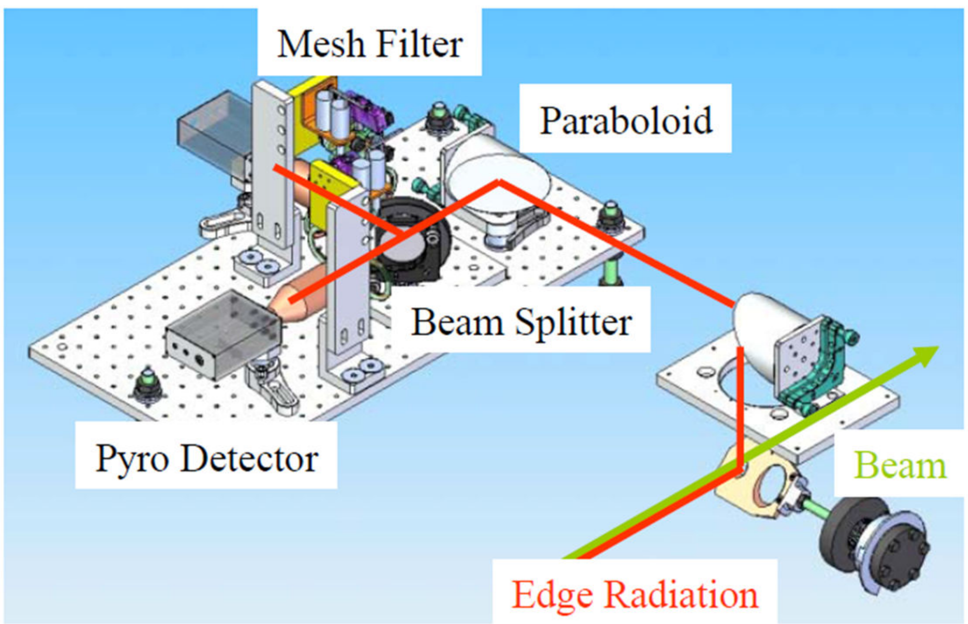
Relative bunch length monitors:

- Integrate **all** coherent power
- Spectrum “DC component” (amplitude) set by *Q*
- Integrated bandwidth increases with shorter bunches
- Specifically*:

$$U\,[\mathrm{mJ}] \approx 14\frac{(Q[\mathrm{nC}])^2}{\Delta t\,[\mathrm{fs}]}$$





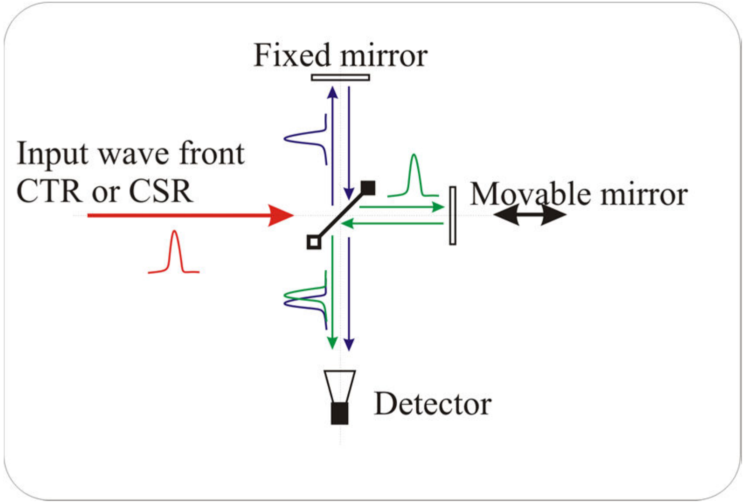


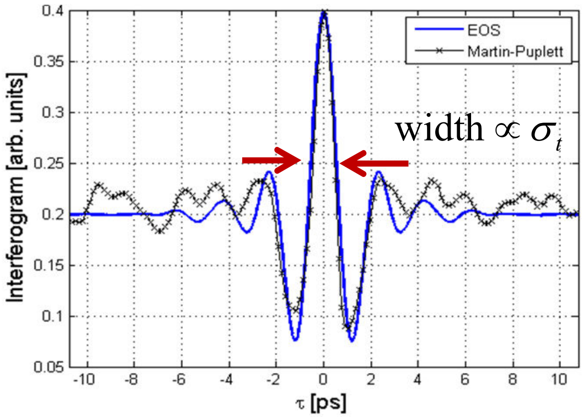
** Relativistic limit, finite imaging system*

“Happek” / Michelson or Martin-Puplett Interferometer

- Thin membrane or wire mesh splitter
- Scanning delay of one arm
- Intensity interferogram yields the profile *autocorrelation function*

$$S(\delta) \equiv \mathrm{Re}\left\{\int E^*(t)E(t+\delta/c)dt\right\} \quad (+\text{ const.})$$



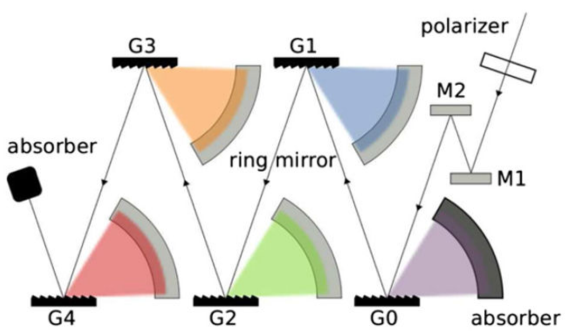


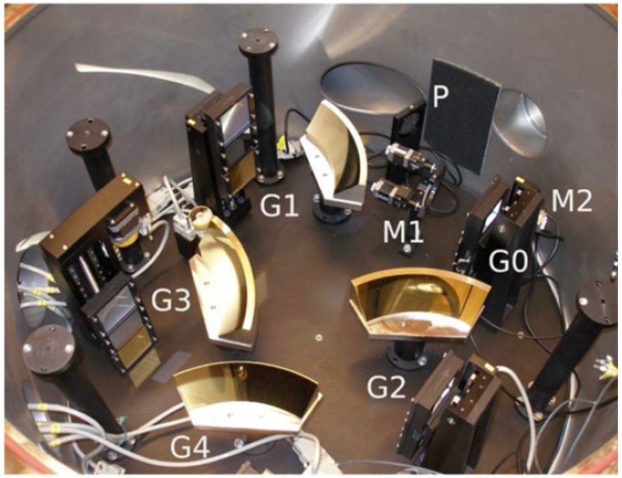
- Equivalent to a *spectrometer*

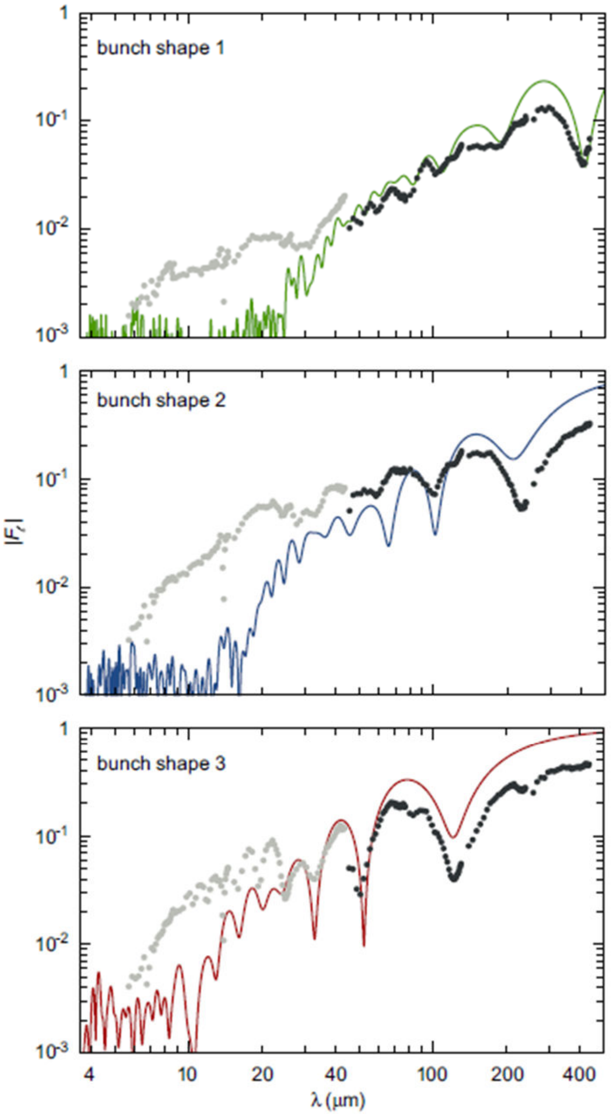
$$S(\delta) \propto \mathrm{Re}\left\{\int |E(\omega)|^2 e^{i\omega\delta/c} d\omega\right\}$$

DESY Cascaded Grating Spectrometer [13]

- Grating-based (high resolution)
- Cascaded
  - Req'd for order sorting
  - 5 – 43 µm or 43 – 435 µm
- Single-shot
- Custom, arc-shaped pyro arrays

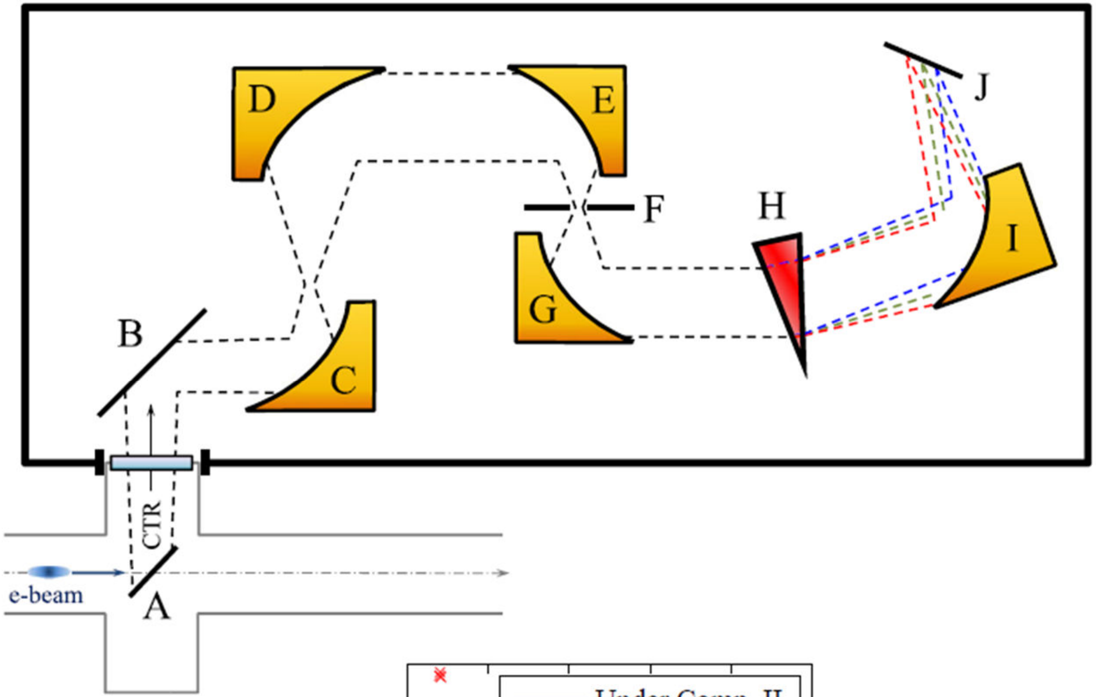


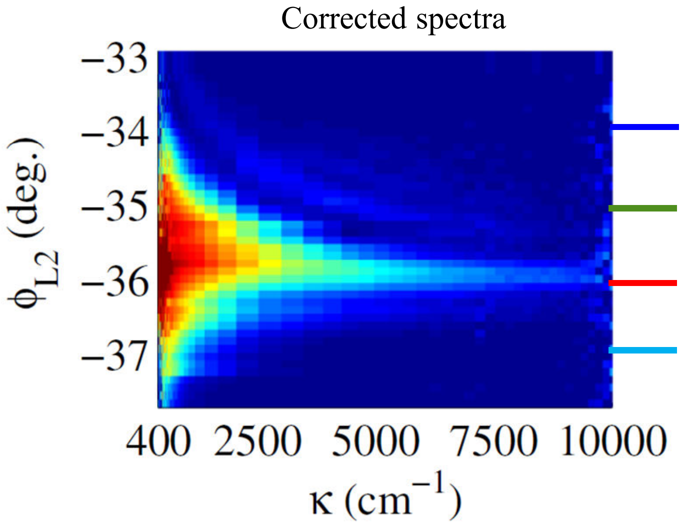


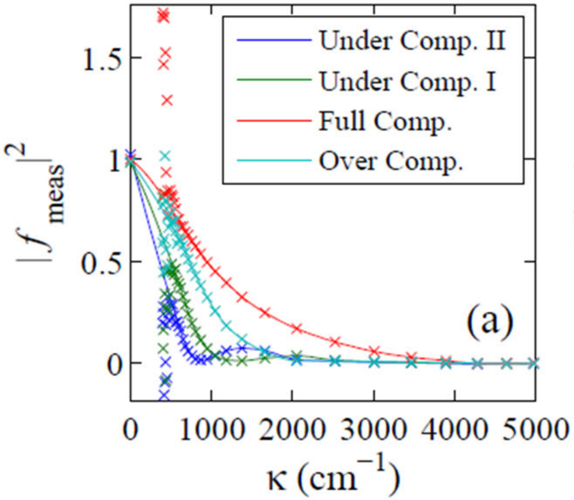


LCLS mid-IR Prism Spectrometer [12]

- mid-IR prism
- Wideband (1 – 40 μm)
- Low resolution
- Linear pyro array
- Single-shot







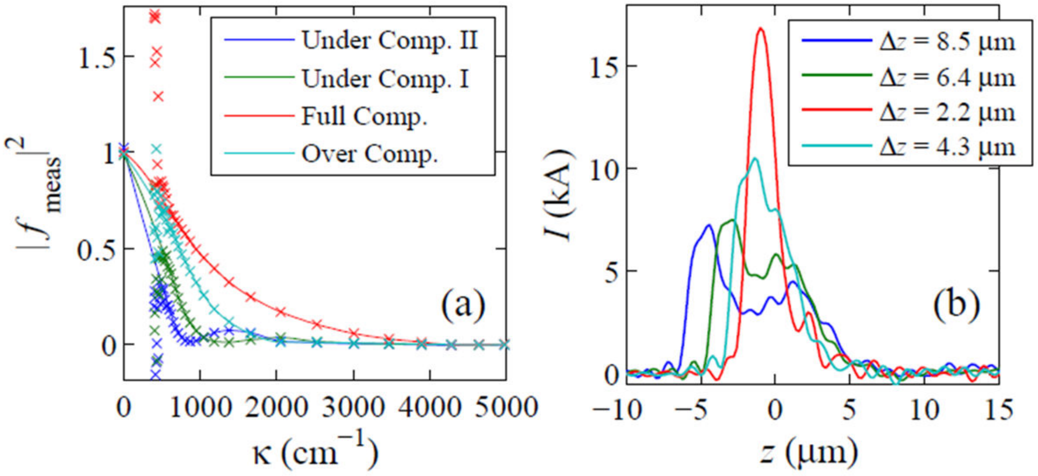
Phase Retrieval: Going the extra mile

SLAC

Can get Kramers-Kronig φ_{min} [14]

$$\phi_{\min}(\omega) = \frac{-\pi}{\omega} P.V.\int_0^\infty d\omega' \frac{\ln\left[|\rho(\omega')|^2 / |\rho(\omega)|^2\right]}{\omega'^2 - \omega^2}$$

- Assumes causal signal
- *Not a unique solution!* [17]
- Compute φ_{min} , invert FT:



Longitudinal Electron Beam Diagnostics

Frequency domain techniques

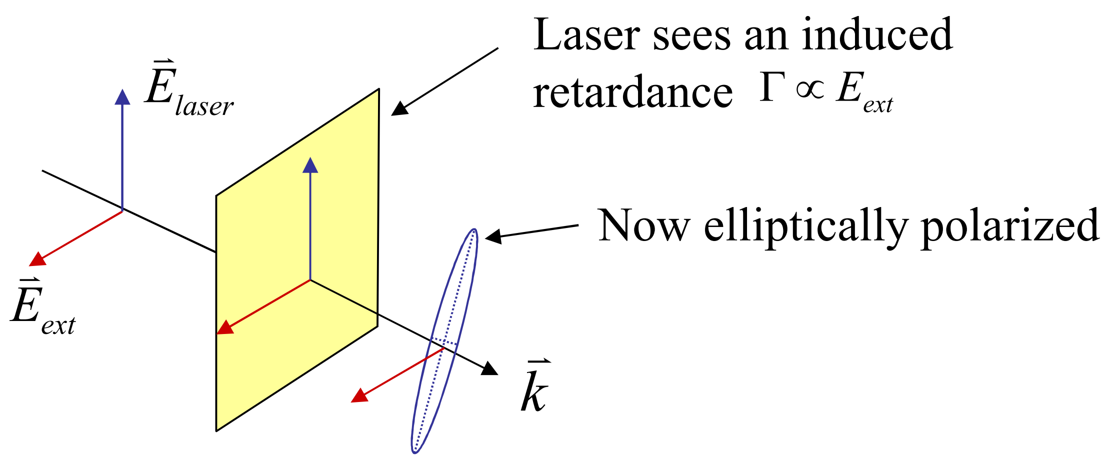
- Measurements of coherent beam radiation spectra
- Interferometry / spectroscopy

Time domain techniques

- Electro-optic sampling
- Streak camera
- Transverse deflecting mode cavities

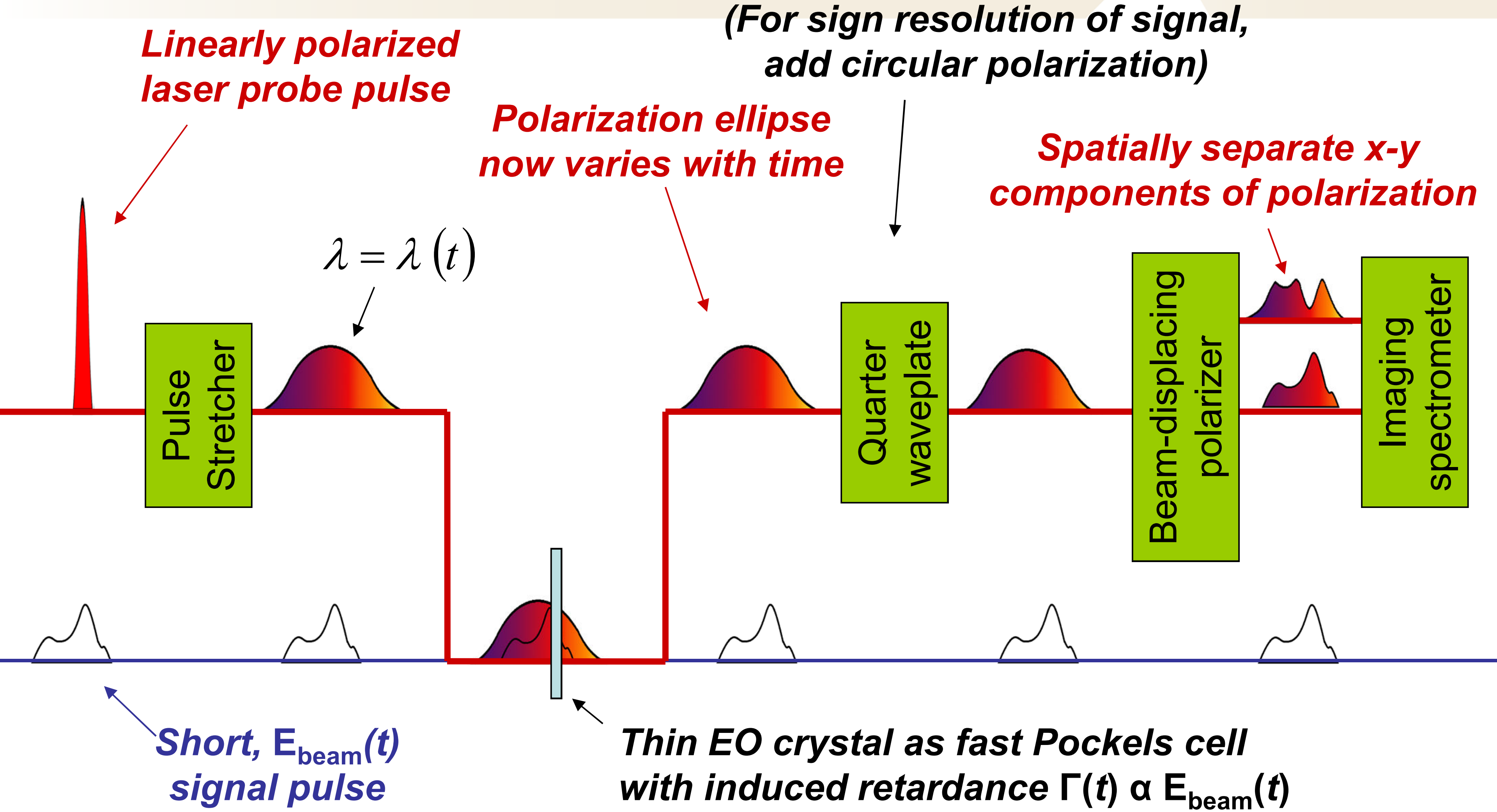
Electro-optic Sampling

- Fast, Pockels-like effect in an electro-optically active crystal



- Thin crystals can “optically switch” at 10s fs
- Radiation response $E_{ext}(t)$
  - Extract coherent beam radiation
  - Direct velocity fields (crystal in vacuum, near beam)
- Analyze changes in probe laser polarization
- Decode a full waveform with a single shot

EO spectral decoding



EO spectral decoding

Polarization analysis – Jones calculus formalism:

Quarter Wave Plate — EO Crystal — Probe Laser

$$\begin{bmatrix} E'_x(t) \\ E'_y(t) \end{bmatrix} = R(45^\circ)\, WP(\pi/2)\, R(-45^\circ)\, R(45^\circ)\, WP[\Gamma(t)]\, R(-45^\circ) \begin{bmatrix} 1 \\ 0 \end{bmatrix} E_L(t)$$

$$\vec{E}'(t) = \begin{bmatrix} 1 + i\, e^{i\Gamma(t)} \\ 1 - i\, e^{i\Gamma(t)} \end{bmatrix} \frac{E_L(t)}{2}$$

- Final BD polarizer separates components, image w/ camera

EO spectral decoding

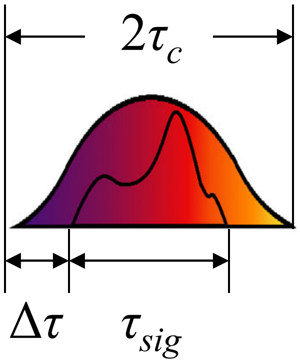
- Resulting temporal intensity profiles:

$$\Rightarrow \begin{cases} I'_x(t) = \frac{1}{2}\{1 - \sin[\Gamma(t)]\} I_L(t) \\ I'_y(t) = \frac{1}{2}\{1 + \sin[\Gamma(t)]\} I_L(t) \end{cases}$$

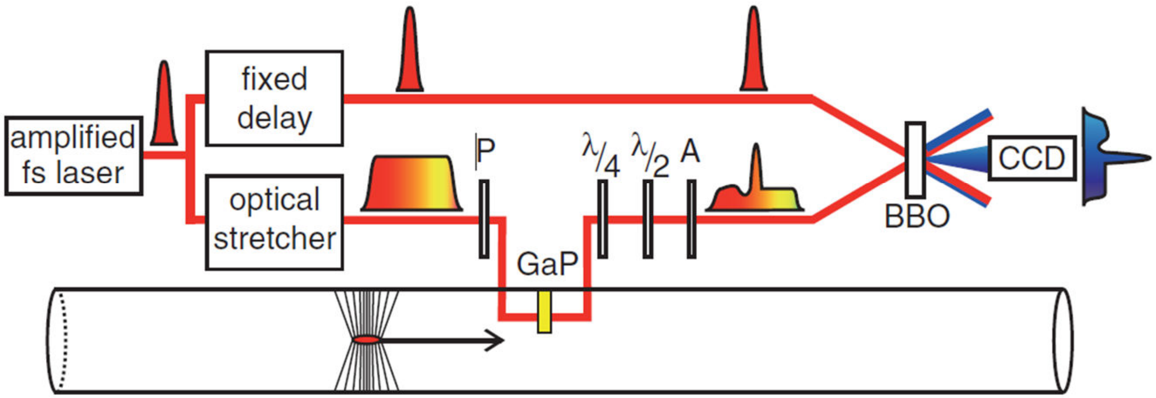
- Difference over sum, small signal limit: $\frac{\Delta}{\Sigma} = \sin[\Gamma(t)] \propto E_{beam}(t)$

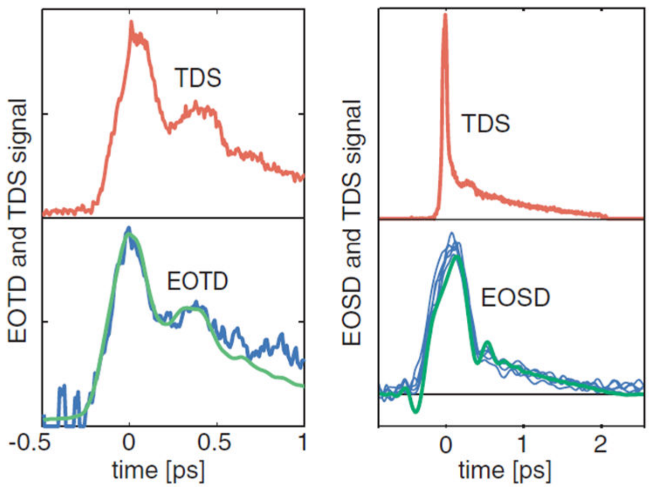
- EOSD "time window" for measurement $\sim 2\tau_c$
  - Minimize for optimum $\boldsymbol{\tau_{res}} \sim (\tau_c\, \tau_0)^{1/2}$
  - τ_0 = 10s fs, τ_c = 1s ps, $\boldsymbol{\tau_{res}} \sim$ 100s fs
  - Lower limit fixed by:

    Timing jitter, signal length, laser BW



EO temporal decoding [18]

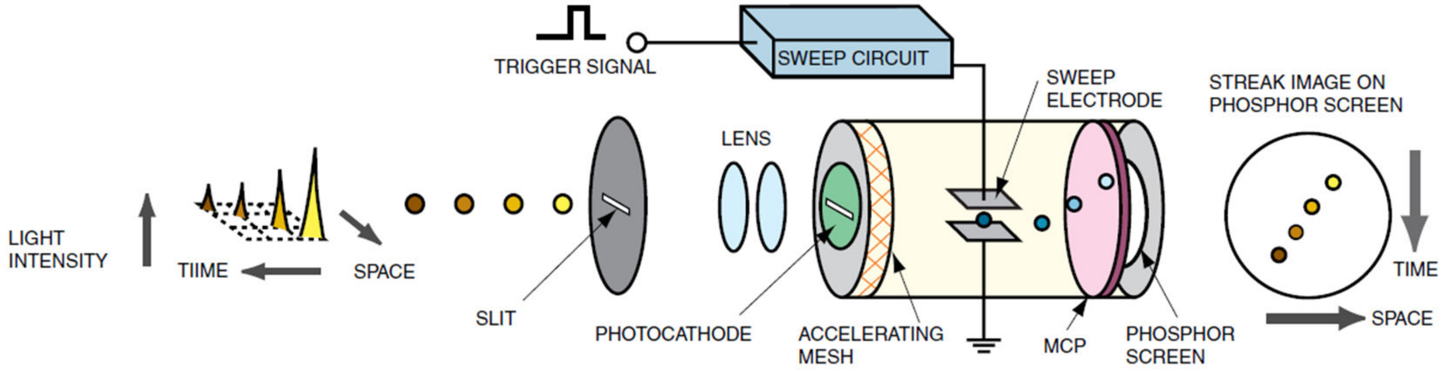




- No chirped duration dependence
- Stronger laser for BBO cross-corr.
- GVD in crystals and cross-corr. res. still limit τ_{res} ~ 50 fs

Streak Camera

- *Incoherent* (high-*f*) radiation pulse sent to streak camera slit
- Streak camera = “keV-scale photoinjector” + deflector + screen [15]

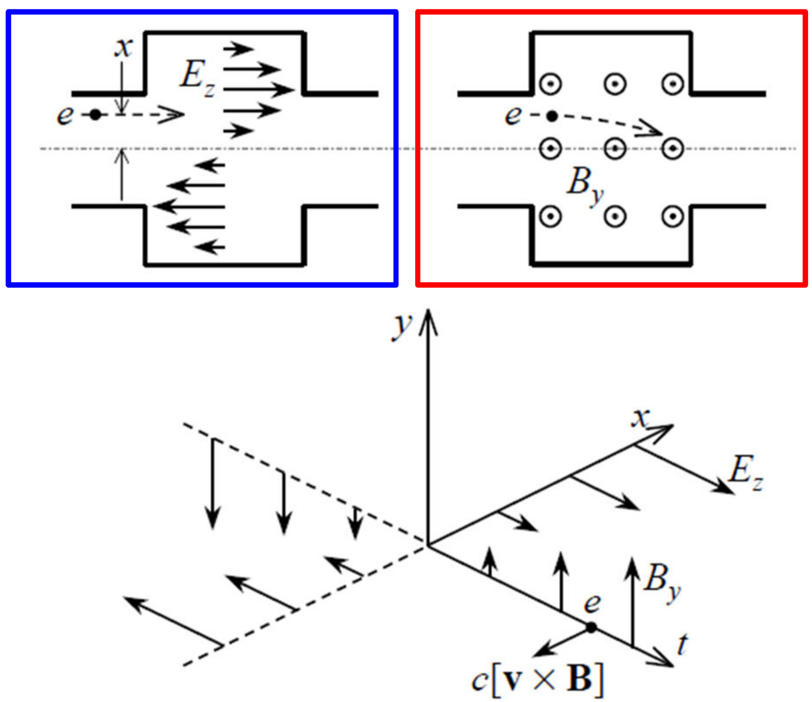


- Fastest units τ_{res} = 200 fs
- Dual sweep benefits high-rate machines [16]
  - One axis still fast, ps sweep
  - Adds slow, perpendicular μs streaking
  - Capture sub-ps resolution of high-rate (MHz) trains

Transvserse TM11 deflecting mode cavity

Dipole mode cavity, 1st order @ zero crossing:

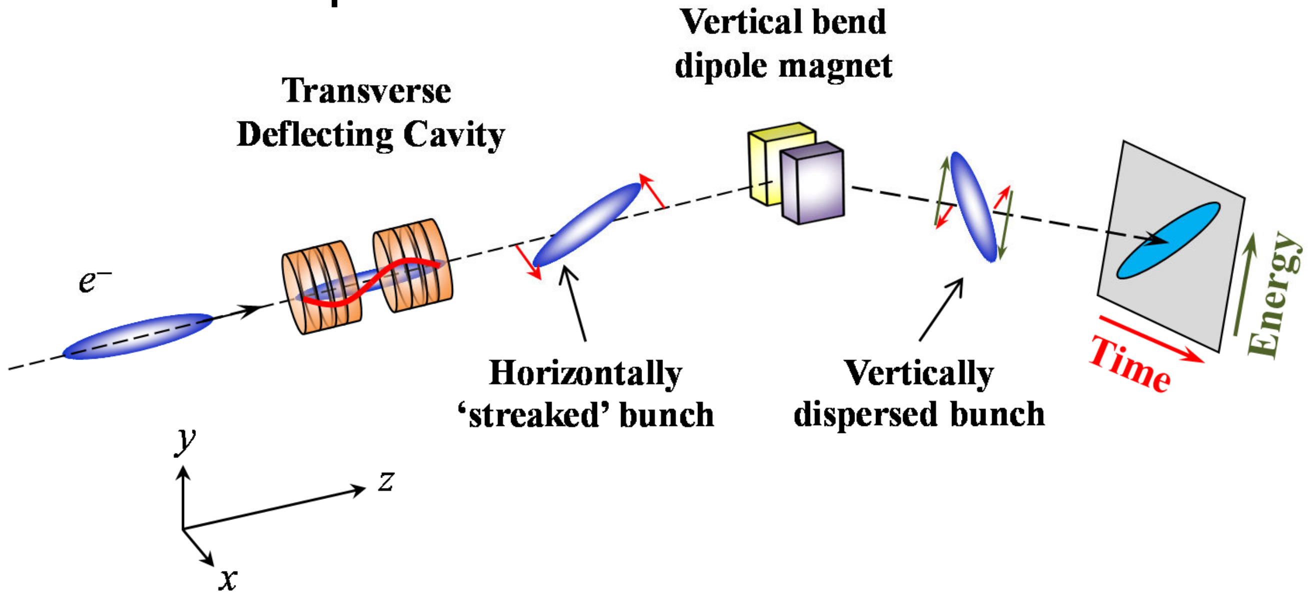
1. *B* introduces $\Delta x'(t)$ ⟵ Deflection
2. *E* introduces $\Delta\delta(x)$



Ref. [19] [20]

Direct *t-E* phase space measurement

Cross deflection with dispersion

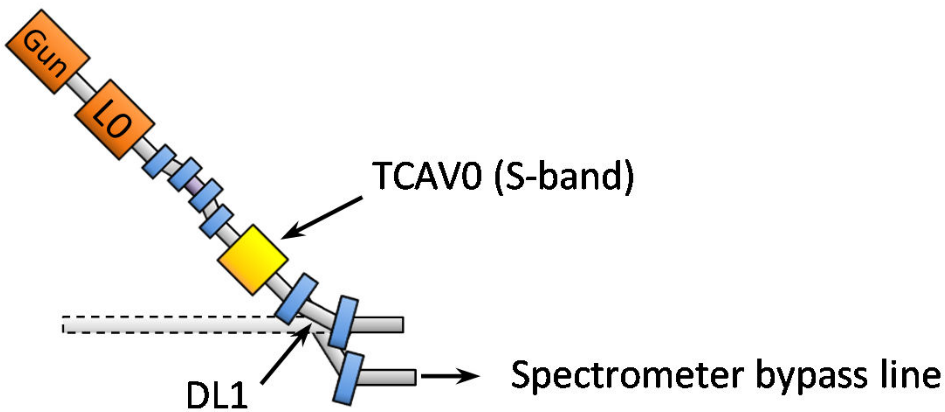


$$\sigma_t = \frac{1}{2\pi f_{\mathrm{rf}}} \frac{E_e}{eV_{\mathrm{rf}}} \sqrt{\frac{\varepsilon_x}{\gamma\beta_x}} \quad \propto \sqrt{E_e} \quad \propto \frac{1}{f_{\mathrm{rf}}}$$

Time resolution | **High-*E* FEL** | **Win at high *f*, SLAC X-band @ 11.4 GHz**

Result:
1 fs rms @ 4 GeV
3 fs rms @ 13 GeV

t-E phase space at LCLS

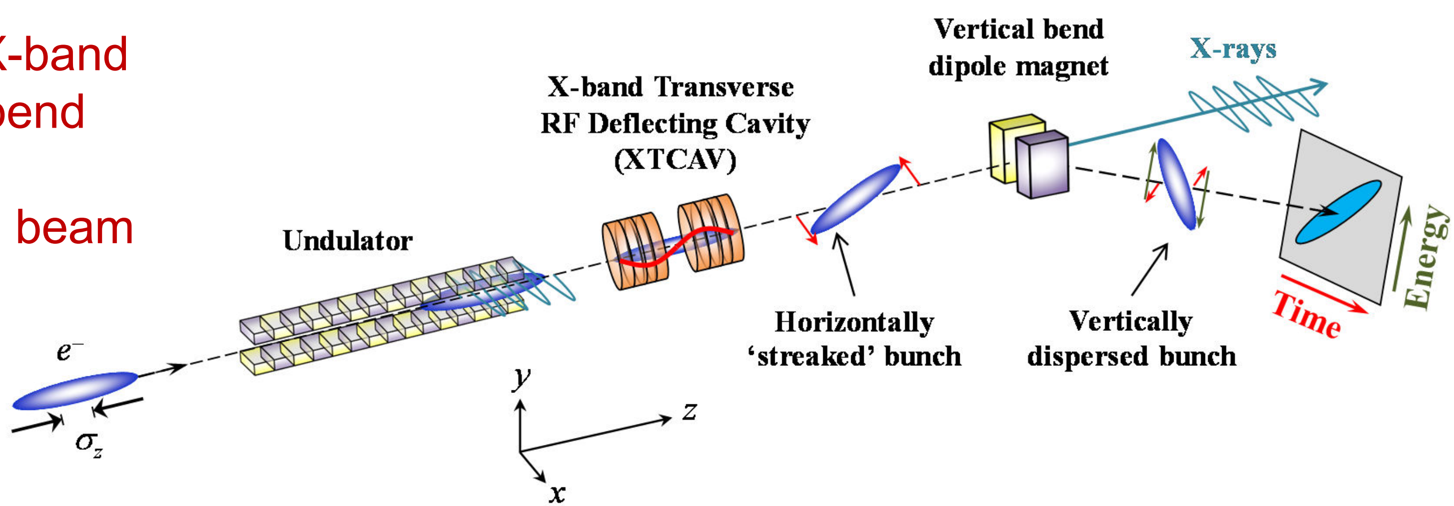


1: Vertical S-band
+ Horiz. Bend

Study injector/LH

2: Horiz. X-band
+ Vert. bend

Study final beam

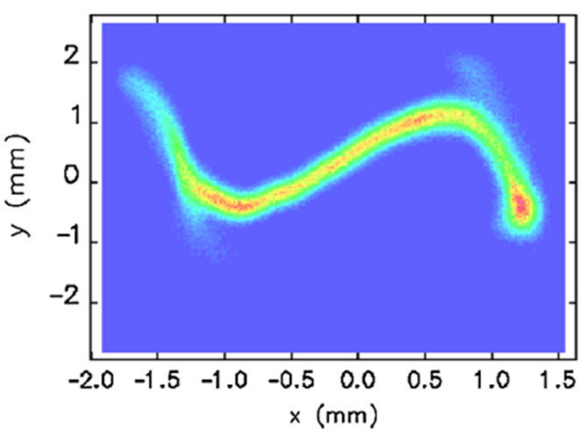


X-ray Pulse Profiling

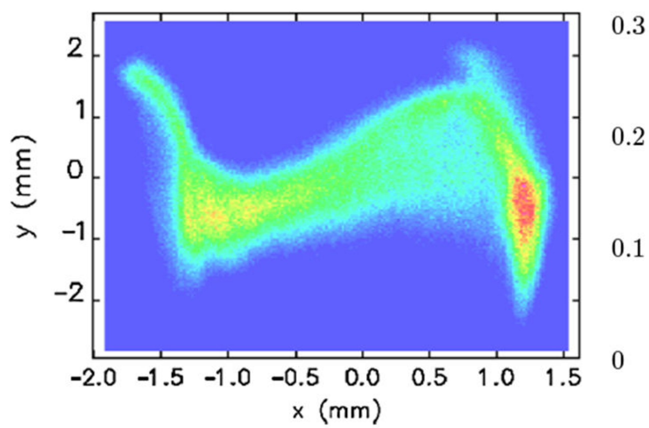
Electron bunch is "parent" of X-ray pulse

Changes in **electron** *t-E* phase space infer resulting **X-ray pulse** profile

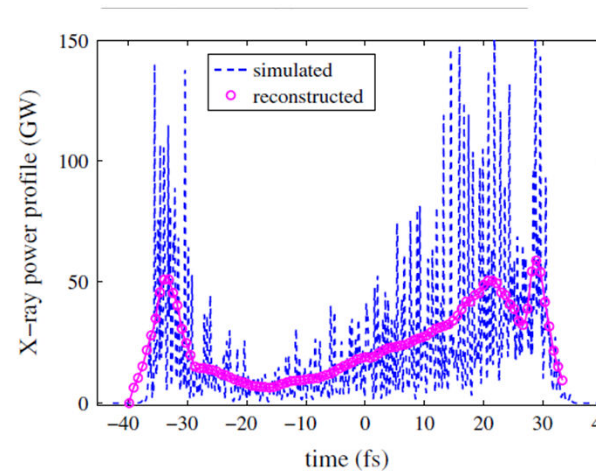
$$P_{\mathrm{FEL}}(t) = [\langle E \rangle_{\mathrm{FEL\ off}}(t) - \langle E \rangle_{\mathrm{FEL\ on}}(t)] \times I(t)$$



FEL off



FEL on



X-ray profile

Proposed: Y. Ding, *et al*, PRST-AB **14**, 120701 (2011)

X-ray Pulse Profiling

Electron bunch is "parent" of X-ray pulse

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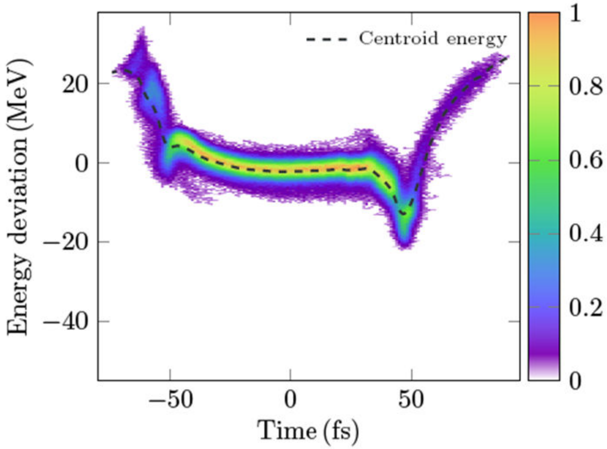
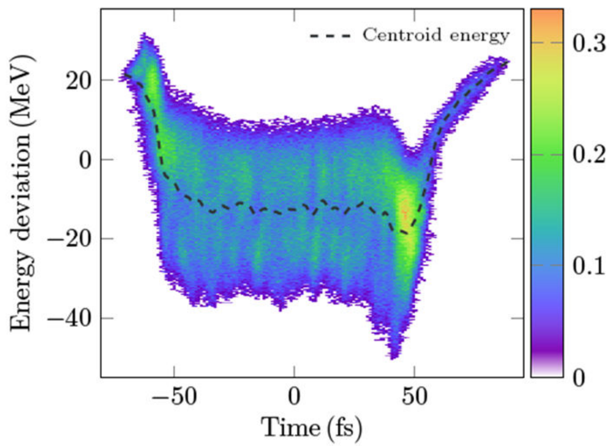
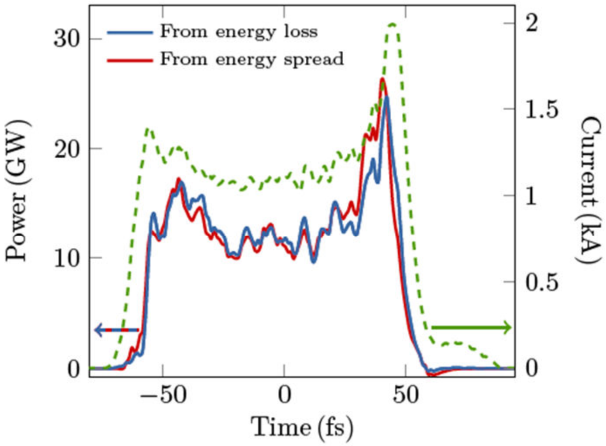


FEL off



FEL on

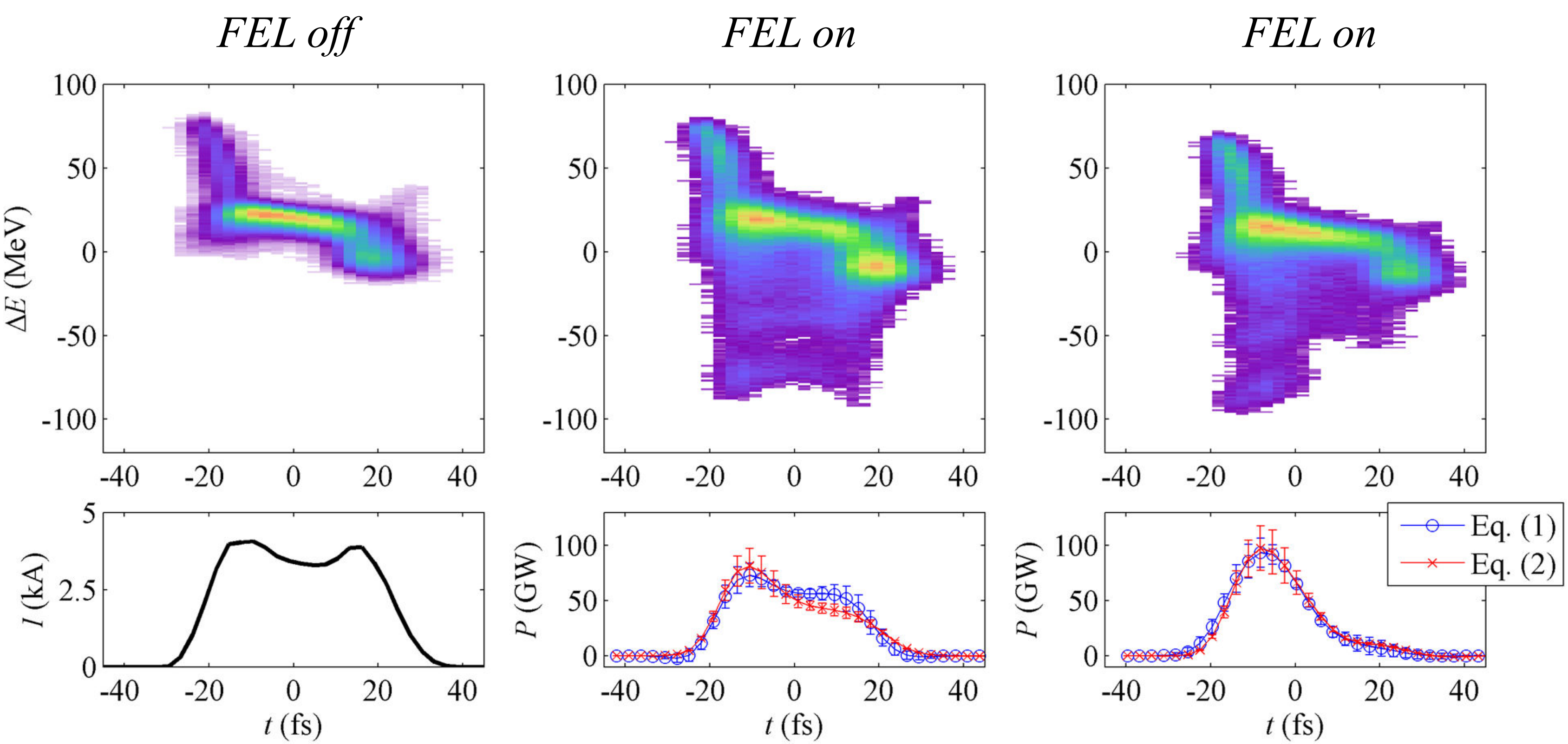


X-ray profile

Proposed: Y. Ding, *et al*, PRST-AB **14**, 120701 (2011)
Demonstrated: C. Behrens, *et al*,, Nat. Comm. **5**, 3762 (2014)

SASE Measurement: Shot-by-shot fluctuations

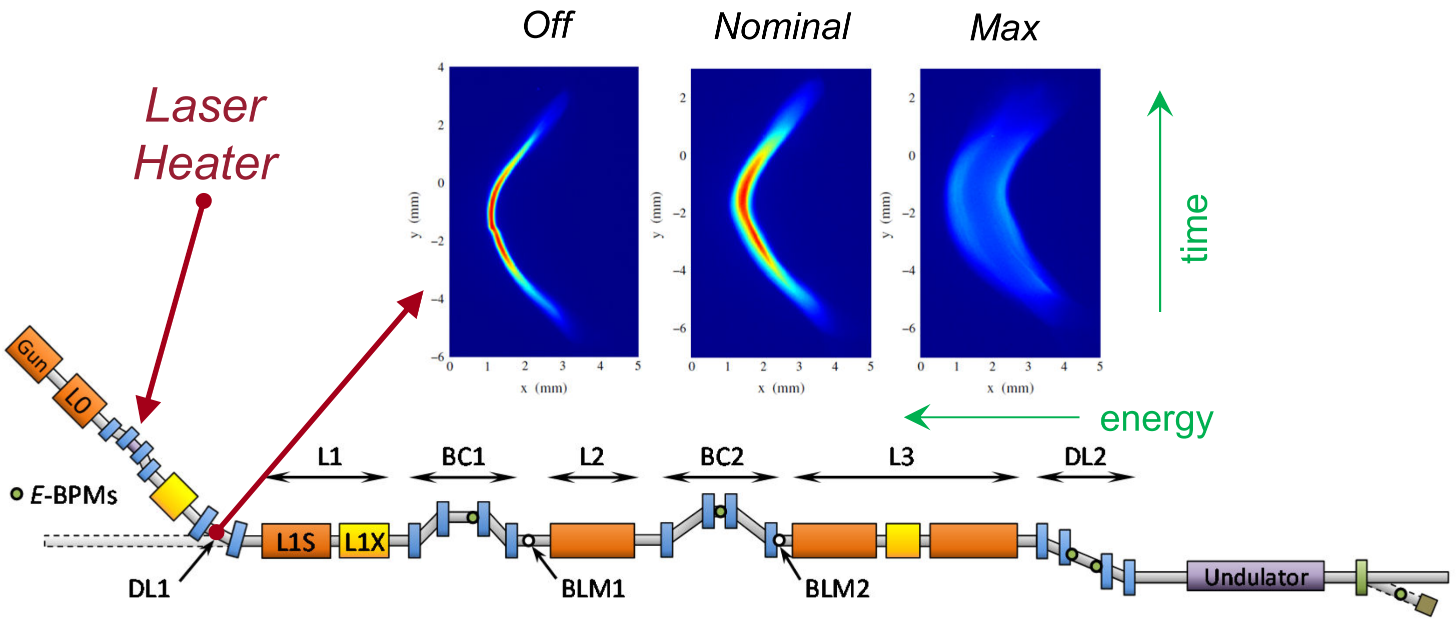
- Uneven or horn lasing identified



The LCLS Laser Heater

Preventing slice energy spread growth (microbunching instability):

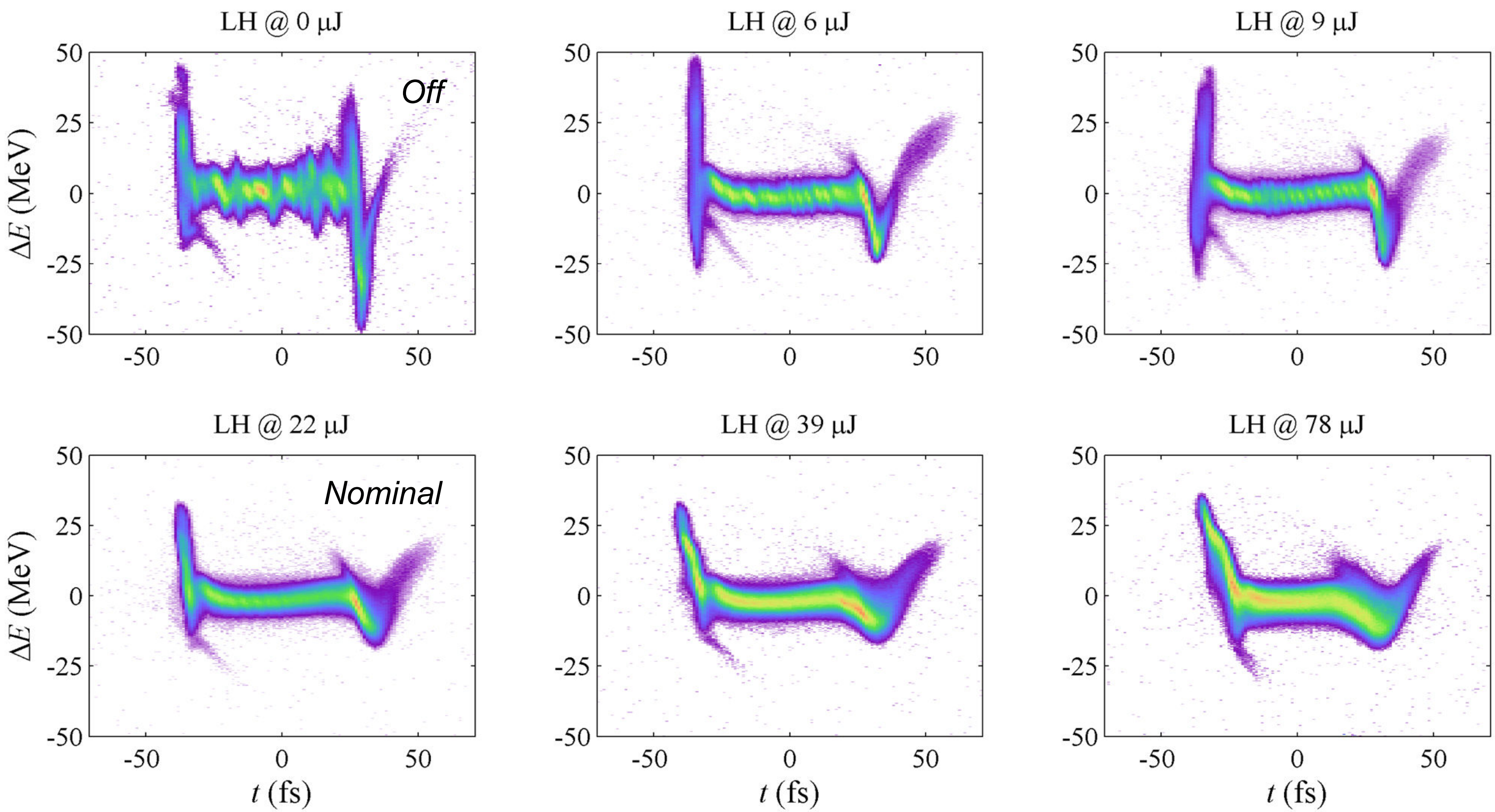
- Increases *E*-spread early, reduce MB gain downstream



- Nominal heating improves LCLS intensity ***20-100%***
- *E*-spread even more critical for harmonic lasing and major challenge for proposed LCLS-II

Z. Huang, *et al*, PRST-AB **7**, 074401 (2004) & Z. Huang, *et al*, PRST-AB **13**, 02073 (2010)

Direct & quantitative study of microbunching



(*Undulators removed for these data*)

Trade offs and challenges

SLAC

Frequency domain	Time domain
• Typically simple passive devices and robust/affordable	• Typically more costly, active devices (lasers, RF structures)
• General approach easily scales to different bunch lengths	• Scaling to very short (fs) bunches challenging
• Loss of phase information, temporal profile ambiguous	• Temporal profile directly, ~no ambiguity in temporal shape

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