EMITTANCE EXCHANGE

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U.S. DEPARTMENT OF ENERGY Office of Science

SLAC NATIONAL ACCELERATOR LABORATORY

CONTENTS

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INTRODUCTION

- Different applications present different requirements on the quality of the particle beam.

- Many beam applications have stringent emittance requirements for successful operation, and are not always compatible with the beam characteristics of available accelerators.

$$\varepsilon_{n,x}\varepsilon_{n,y}\varepsilon_{n,z} \qquad\qquad \varepsilon_{n,x} : \varepsilon_{n,y} : \varepsilon_{n,z}$$

- In this case it will be very useful to exchange the emittances between two different dimensions to make all of the requirements about emittance are met.

PHYSICAL ANALYSIS

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- Uncoupled beam sigma matrix:

$$\boldsymbol{\sigma}_0 = \begin{pmatrix} \boldsymbol{\sigma}_x & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}_z \end{pmatrix} = \begin{pmatrix} \varepsilon_{x0}\beta_x & -\varepsilon_{x0}\alpha_x & 0 & 0 \\ -\varepsilon_{x0}\alpha_x & \varepsilon_{x0}\gamma_x & 0 & 0 \\ 0 & 0 & \varepsilon_{z0}\beta_z & -\varepsilon_{z0}\alpha_z \\ 0 & 0 & -\varepsilon_{z0}\alpha_z & \varepsilon_{z0}\gamma_z \end{pmatrix}$$

- Through the transfer matrix: $\boldsymbol{\sigma} = \mathbf{R}\boldsymbol{\sigma}_0\mathbf{R}^{\mathrm{T}}$

- The transfer matrix should be coupled to achieve the emittance exchange.

- Coupled component: dipole, dogleg, chicane, etc.

PHYSICAL ANALYSIS

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- The 4*4 matrix is constructed from four 2*2 blocks,

$$\mathbf{R} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}$$

- The beam matrix can be expressed as:

$$\boldsymbol{\sigma} = \begin{pmatrix} \mathbf{A}\boldsymbol{\sigma}_{\mathbf{x}}\mathbf{A}^{\mathbf{T}} + \mathbf{B}\boldsymbol{\sigma}_{\mathbf{z}}\mathbf{B}^{\mathbf{T}} & \mathbf{A}\boldsymbol{\sigma}_{\mathbf{x}}\mathbf{C}^{\mathbf{T}} + \mathbf{B}\boldsymbol{\sigma}_{\mathbf{z}}\mathbf{D}^{\mathbf{T}} \\ \mathbf{C}\boldsymbol{\sigma}_{\mathbf{x}}\mathbf{A}^{\mathbf{T}} + \mathbf{D}\boldsymbol{\sigma}_{\mathbf{z}}\mathbf{B}^{\mathbf{T}} & \mathbf{C}\boldsymbol{\sigma}_{\mathbf{x}}\mathbf{C}^{\mathbf{T}} + \mathbf{D}\boldsymbol{\sigma}_{\mathbf{z}}\mathbf{D}^{\mathbf{T}} \end{pmatrix}$$

- Final emittance after matrix transformation

$$\varepsilon_x{}^2 = \left| \mathbf{A}\boldsymbol{\sigma}_{\mathbf{x}}\mathbf{A}^{\mathbf{T}} + \mathbf{B}\boldsymbol{\sigma}_{\mathbf{z}}\mathbf{B}^{\mathbf{T}} \right|$$

$$\varepsilon_z{}^2 = \left| \mathbf{C}\boldsymbol{\sigma}_x\mathbf{C}^{\mathbf{T}} + \mathbf{D}\boldsymbol{\sigma}_{\mathbf{z}}\mathbf{D}^{\mathbf{T}} \right|$$

PHYSICAL ANALYSIS

$$\varepsilon_x{}^2 = |\mathbf{A}|^2 \varepsilon_{x0}{}^2 + (1-|\mathbf{A}|)^2 \varepsilon_{z0}{}^2 + \varepsilon_{x0}\varepsilon_{z0}\lambda^2$$

$$\varepsilon_z{}^2 = (1-|\mathbf{A}|)^2 \varepsilon_{x0}{}^2 + |\mathbf{A}|^2 \varepsilon_{z0}{}^2 + \varepsilon_{x0}\varepsilon_{z0}\lambda^2$$

$$\boldsymbol{\sigma}_{\mathbf{x}} = \varepsilon_{x0}\mathbf{Q}_{\mathbf{x}}\mathbf{Q}_{\mathbf{x}}{}^{\mathbf{T}} \qquad \mathbf{Q}_{\mathbf{x}} = \frac{1}{\sqrt{\beta_x}}\begin{pmatrix} \beta_x & 0 \\ -\alpha_x & 1 \end{pmatrix}$$

$$\boldsymbol{\sigma}_{\mathbf{z}} = \varepsilon_{z0}\mathbf{Q}_{\mathbf{z}}\mathbf{Q}_{\mathbf{z}}{}^{\mathbf{T}} \qquad \mathbf{Q}_{\mathbf{z}} = \frac{1}{\sqrt{\beta_z}}\begin{pmatrix} \beta_z & 0 \\ -\alpha_z & 1 \end{pmatrix}$$

$$\mathbf{U} = \mathbf{Q}_{\mathbf{x}}{}^{-1}\mathbf{A}^{a}\mathbf{B}\mathbf{Q}_{\mathbf{z}}$$

$$\mathbf{V} = \mathbf{Q}_{\mathbf{x}}{}^{-1}\mathbf{C}^{\mathbf{a}}\mathbf{D}\mathbf{Q}_{\mathbf{z}}$$

$$\mathbf{tr}\{\mathbf{U}\mathbf{U}^{\mathbf{T}}\} = \mathbf{tr}\{\mathbf{V}\mathbf{V}^{\mathbf{T}}\} = \lambda^2 \geq 0$$

- To achieve $\varepsilon_x = \varepsilon_{z0}$ and $\varepsilon_z = \varepsilon_{x0}$

$$|\mathbf{A}| = 0$$

$$\lambda^2 = 0 \quad \Rightarrow \quad A_{i,j} = D_{i,j} = 0$$

$$\mathbf{R} = \begin{pmatrix} 0 & 0 & * & * \\ 0 & 0 & * & * \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{pmatrix}$$

Reference: Cornacchia M, Emma P. PRST, 2002, 5(8): 084001.

BEAMLINE DESIGN

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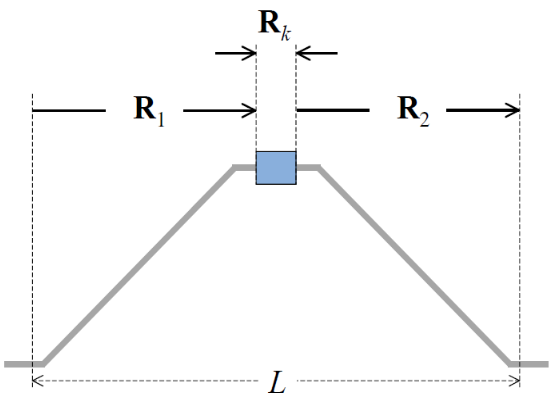
1



$$\mathbf{R}_k = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{R_1} = \begin{pmatrix} 1 & L/2 & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & \eta & 1 & \xi/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{R}_2 = \begin{pmatrix} 1 & L/2 & 0 & -\eta \\ 0 & 1 & 0 & 0 \\ 0 & -\eta & 1 & \xi/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$k \equiv \frac{eV_0}{aE}$$

η and ξ are dispersion and momentum compaction Factor respectively

$$\mathbf{R} = \begin{pmatrix} 1-\eta k & L & kL/2 & k(\frac{\xi L}{4}-\eta^2) \\ 0 & 1+\eta k & k & k\xi/2 \\ k\xi/2 & k(\frac{\xi L}{4}-\eta^2) & 1-\eta k & \xi \\ k & kL/2 & 0 & 1+\eta k \end{pmatrix}$$

$$\mathbf{R} = \begin{pmatrix} 0 & 0 & * & * \\ 0 & 0 & * & * \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{pmatrix}$$

- Considering a simple case: $1-\eta k = 0$ → $|\mathbf{A}| = |\mathbf{D}| = 0$ $\quad |\mathbf{B}| = |\mathbf{C}| = 1$

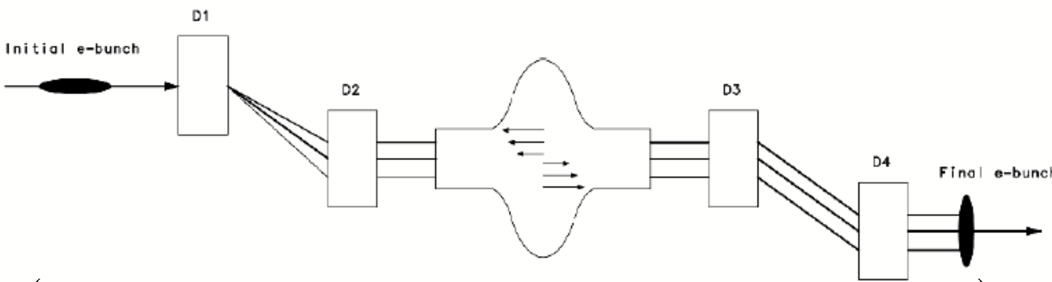
$$\varepsilon_x = \sqrt{\varepsilon_{z0}{}^2 + 4\sigma_{x'}{}^2\sigma_\delta{}^2\eta^2}$$

$$\varepsilon_z = \sqrt{\varepsilon_{x0}{}^2 + 4\sigma_{x'}{}^2\sigma_\delta{}^2\eta^2}$$

Reference: Cornacchia M, Emma P. PRST, 2002, 5(8): 084001.

BEAMLINE DESIGN

2 An improved beamline



$$\mathbf{R}_k = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{pmatrix}$$

$$\mathbf{R} = \begin{pmatrix} 1+\eta k & L(1+\eta k) & kL/2 & k(\frac{\xi L}{4}+\eta^2)+2\eta \\ 0 & 1+\eta k & k & k\xi/2 \\ k\xi/2 & k(\frac{\xi L}{4}+\eta^2)+2\eta & 1+\eta k & \xi+\eta k\xi \\ k & kL/2 & 0 & 1+\eta k \end{pmatrix}$$

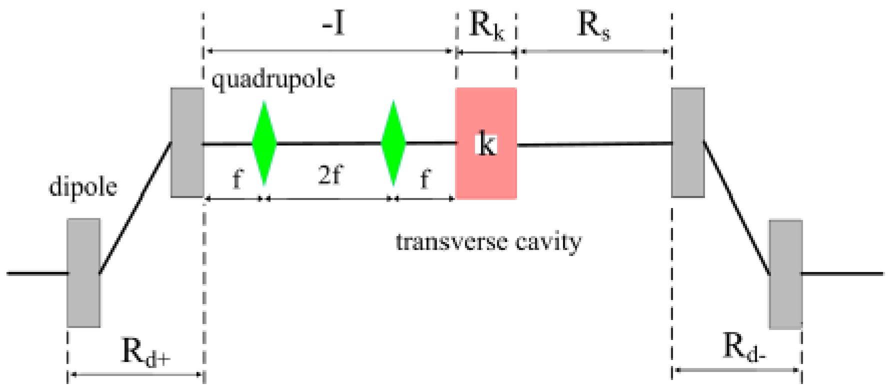
$$\mathbf{R}_2 = \mathbf{R}_1 = \begin{pmatrix} 1 & L/2 & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & \eta & 1 & \xi/2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Considering the case $1+\eta k = 0$

$$\mathbf{R} = \begin{pmatrix} 0 & 0 & kL/2 & \frac{k\xi L}{4}+\eta \\ 0 & 0 & k & k\xi/2 \\ k\xi/2 & \frac{k\xi L}{4}+\eta & 0 & 0 \\ k & kL/2 & 0 & 0 \end{pmatrix} \Longleftrightarrow \mathbf{R} = \begin{pmatrix} 0 & 0 & * & * \\ 0 & 0 & * & * \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{pmatrix}$$

Reference: Emma P, Huang Z, Kim K J, et al. PRST, 2006, 9(10): 100702.

3 Another improved beamline



- Considering the case $\eta k = 1$

$$R_c = \begin{bmatrix} 0 & 0 & k(L+S) & k\xi(L+S)-\eta \\ 0 & 0 & k & k\xi \\ -k\xi & \eta - kL\xi & 0 & 0 \\ -k & -kL & 0 & 0 \end{bmatrix}$$

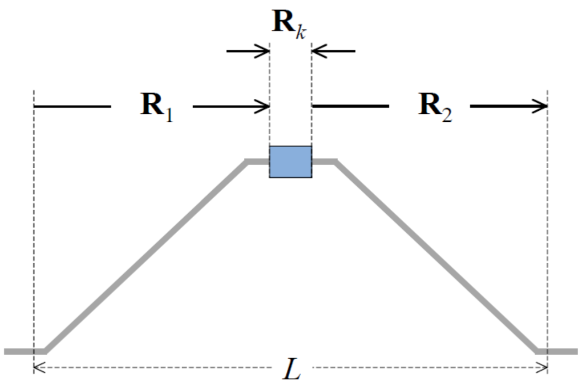
⟺

$$\mathbf{R} = \begin{pmatrix} 0 & 0 & * & * \\ 0 & 0 & * & * \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{pmatrix}$$

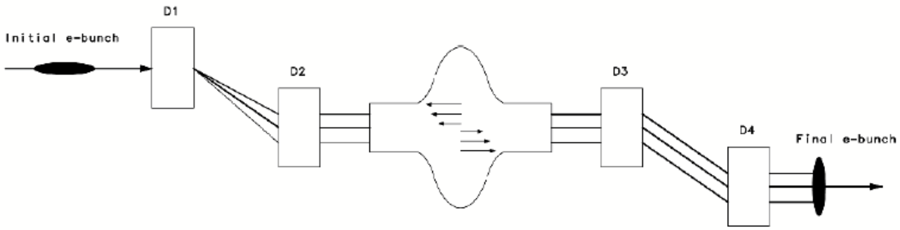
Reference: Xiang D, Chao A. PRST, 2011, 14(11): 114001.

BEAMLINE DESIGN

- Comparison of different beamlines

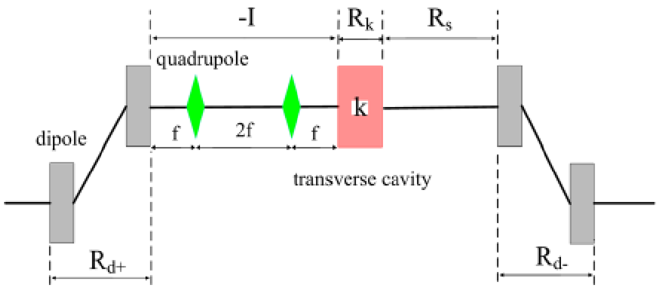


uncomplete exchange



complete exchange

Experimental verification
offset of the beam trajectory



complete exchange
no offset of the beam trajectory
paperwork

APPLICATIONS

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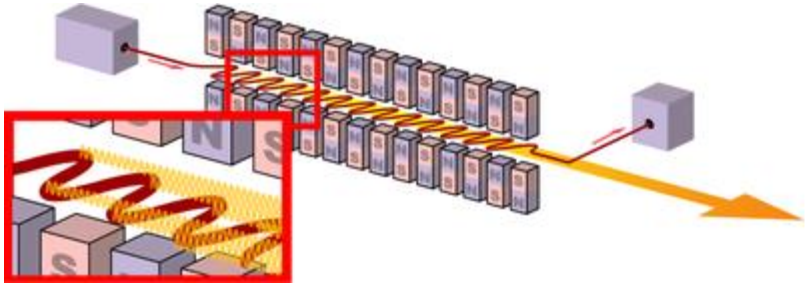
✓ **Emittance Exchange**

- **To achieve lower transverse emittance in FELs.**

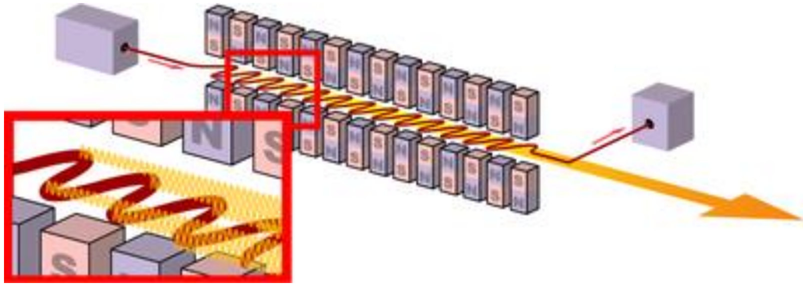


- **Phase space exchange (generating tunable subpicosecond electron-bunch-train).**

APPLICATIONS

- **To achieve lower transverse emittance in FELs**

Transverse emittance $\varepsilon_{n,x} \leq \lambda\gamma / 4\pi$



$(\varepsilon_x, \varepsilon_y, \varepsilon_z) = (1,1,0.1)$

round to flat beam transformation

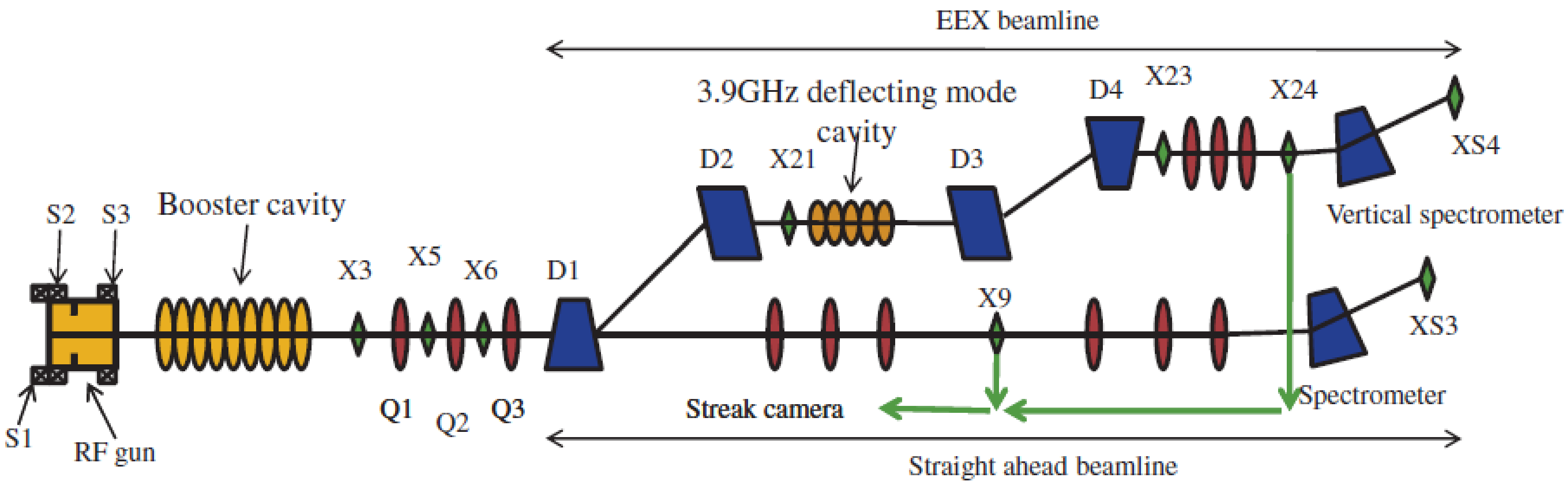
$(10,0.1,0.1)$

longitudinal to transverse emittance exchange

$(0.1,0.1,10)$

APPLICATIONS

- **To achieve lower transverse emittance in FELs**



	Simulated		Measured	
	In	Out	In	Out
ε_{nx}	2.9	13.2	2.9 ± 0.1	11.3 ± 1.1
ε_{ny}	2.4	2.4	2.4 ± 0.1	2.9 ± 0.5
ε_{nz}	13.1	3.2	13.1 ± 1.3	3.1 ± 0.3

Reference: Ruan J, Johnson A S, Lumpkin A H, et al. PRL, 2011, 106(24): 244801.

APPLICATIONS

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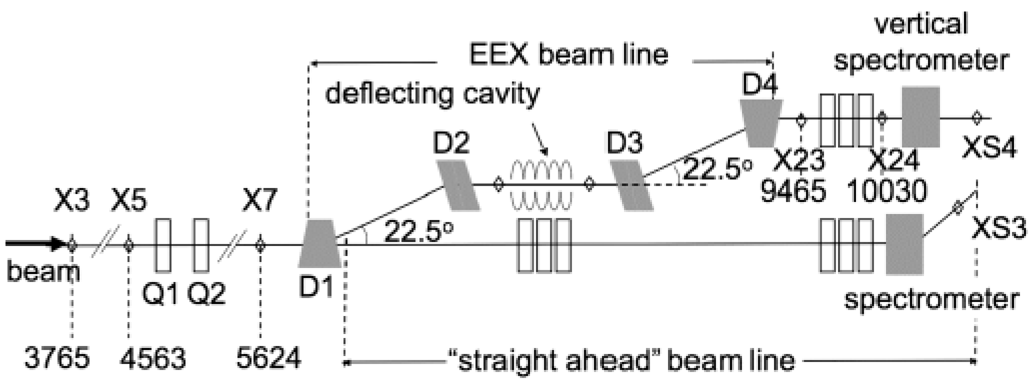
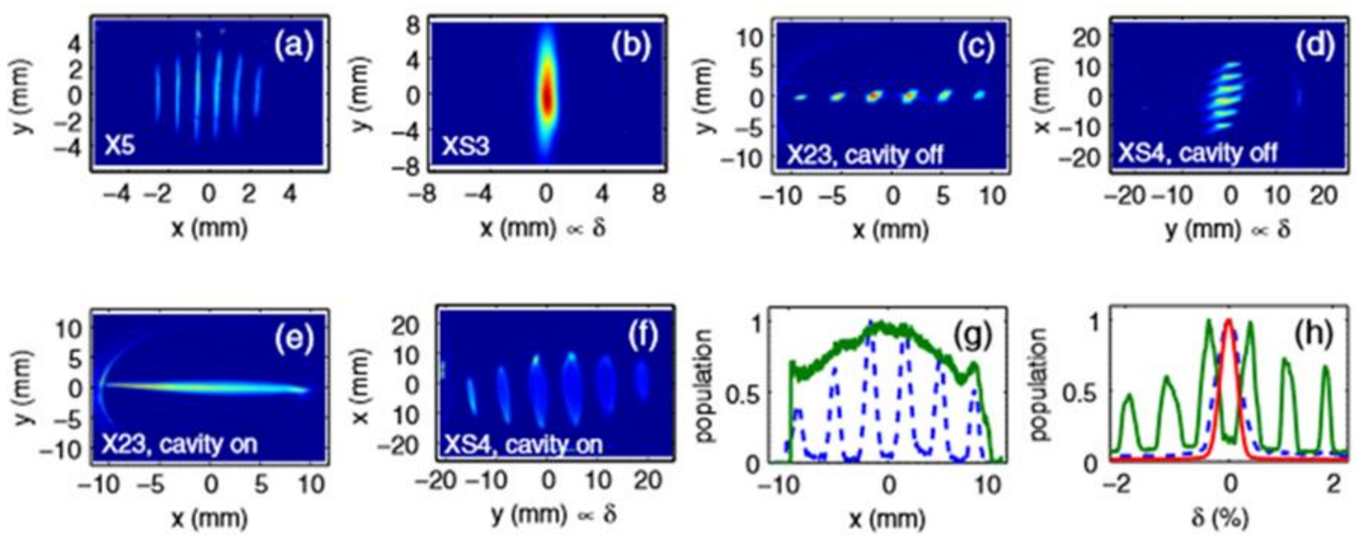
- Tunable Subpicosecond Electron-Bunch-Train Generation



Result:

Bunch spacing: 1.2ps
rms duration for each bunch: <300fs

Bunch charge:
slits out: 550pC
slits in: ~15pC



1、generation of super-radiant radiation
2、excitation of wakefields in beam-driven acceleration

Reference: Sun Y E, Piot P, Johnson A, et al. PRL, 2010, 105(23): 234801.

POSSIBLE LIMITATIONS

- EEX is expensive due to RF and thermal considerations
- Modern photoinjectors produce low emittance beam (no need for EEX)

SUMMARY

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- The physical principle

  Emittance exchange condition:

$$\mathbf{R}=\begin{pmatrix} 0 & 0 & * & * \\ 0 & 0 & * & * \\ * & * & 0 & 0 \\ * & * & 0 & 0 \end{pmatrix}$$

- Three different beamlines

  Advantages and disadvantages

- Applications

  1 To achieve lower transverse emittance in FELs

  2 Phase space exchange to generate tunable subpicosecond electron-bunch-train

REFERENCES

[1] Cornacchia M, Emma P. PRST, 2002, 5(8): 084001.

[2] Emma P, Huang Z, Kim K J, et al. PRST, 2006, 9(10): 100702.

[3] Xiang D, Chao A. PRST, 2011, 14(11): 114001.

[4] Ruan J, Johnson A S, Lumpkin A H, et al. PRL, 2011, 106(24): 244801.

[5] Sun Y E, Piot P, Johnson A, et al. PRL, 2010, 105(23): 234801.

Thank you