Coherent synchrotron radiation in magnetic bunch compressors

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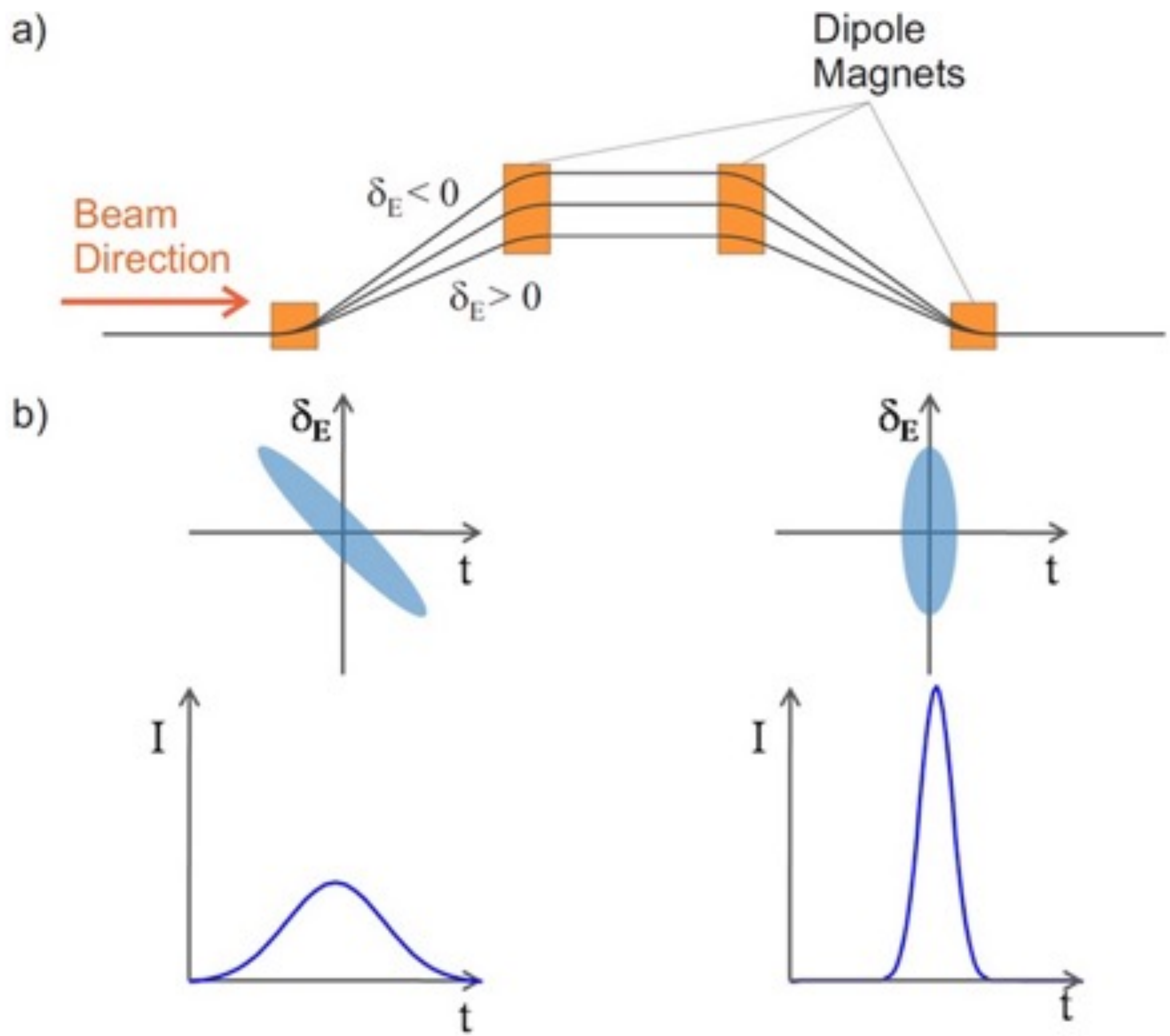
Outline

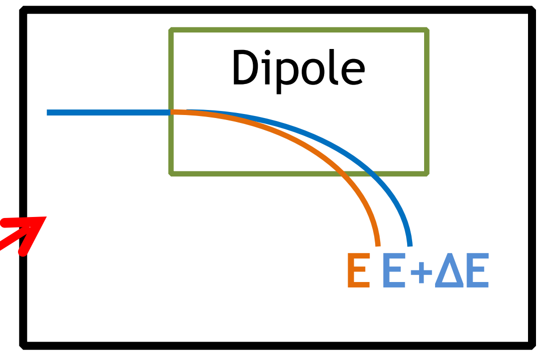
- Magnetic bunch compressor
- Coherent synchrotron radiation (CSR)
  - Background
  - Theory
- Effects on emittance in bunch compressor chicanes
  - Projected emittance growth
- Techniques to mitigate CSR effects

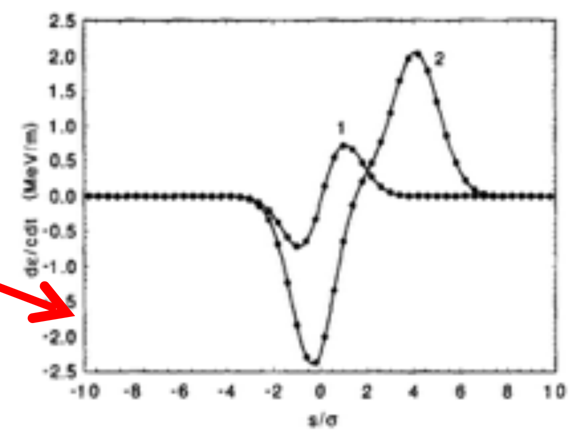
- Particles with different energies will take different paths
- The energy dispersion in the bunch will lead to a change in particles' relative positions.



Coherent Synchrotron Radiation

- Charged particles traveling in an arc emit radiation
- This is usually good (indeed, without it we wouldn't have light sources…) but in terms of beam dynamics:
  - Electrons emitting radiation lose energy, and in a dispersive system (eg. dipole), this will affect the trajectory
  - In a bunch, electrons can reabsorb some of the radiation emitted by other electrons, changing the energy distribution (some electrons are losing energy and some are gaining)

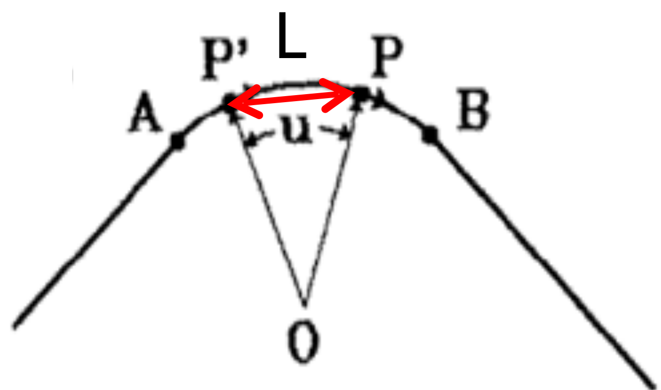




Instantaneous change in electron energy vs distance along bunch [1]

Coherent Synchrotron Radiation

- First step: Solve for the fields of an electron in a bend
- The electric fields of an electron at point P', evaluated at a point P, can be described by [1]:



Centripetal Acceleration: $\frac{v^2}{R}$

Coulomb Term

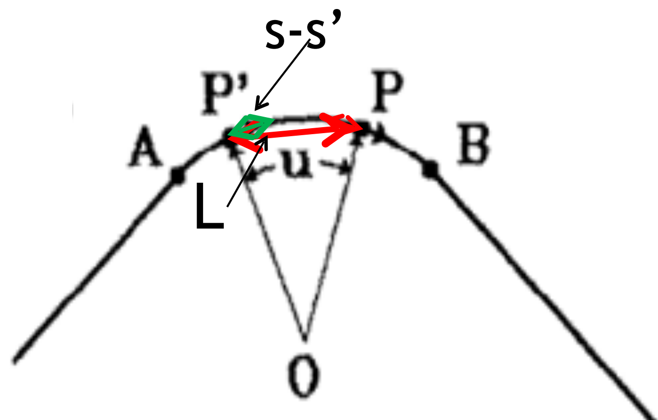
$$E(P) = \frac{e}{\gamma^2}\frac{(\boldsymbol{n} - \boldsymbol{\beta}')}{L^2(1 - \boldsymbol{n}\cdot\boldsymbol{\beta}')^3} + \frac{e}{c^2}\frac{\boldsymbol{n}\times[(\boldsymbol{n} - \boldsymbol{\beta}')\times\boldsymbol{g}']}{L(1 - \boldsymbol{n}\cdot\boldsymbol{\beta}')^3}$$

$\sqrt{1 - \frac{1}{\beta'^2}}$ $\frac{v'}{c}$ $\frac{\vec{v}}{v}$

Radiation Term

Coherent Synchrotron Radiation

- Note: Fields originating from electron at P' at time t' will arrive at P at a retarded time t.
- If two electrons are separated by distance s-s', the radiation from s' will be felt at a later time by the electron at s after the front electron traverses the angle u-Δs and the fields travel the distance L:



$$(s - s') = (1 - \beta)Ru + \frac{Ru^3}{24}$$

Slippage length: Distance between arc length and chord length from P' to P

Due to difference in speed between electrons and radiation

Coherent Synchrotron Radiation

- We can solve the fields analytically for two electrons, then calculate the change in energy:
$\frac{d\mathcal{E}}{cdt} = \vec{F} \cdot \vec{\beta} = e\vec{E} \cdot \vec{\beta}$
- By convolving this solution with the longitudinal charge density, λ(*s*') we can obtain the cumulative forces acting on an electron at *s*
- To obtain the total change in energy over a dipole, one then integrates the instantaneous energy change over the entire bunch trajectory
  – Not trivial! Interaction length scales as $\sigma_z \gamma^2$ and can easily extend 10-20m after the bend.

Coherent Synchrotron Radiation

- For the most relevant case, a Gaussian distribution, a long bunch and a longer magnet, ie when:

$$\frac{1}{\gamma} \ll \left(\frac{24 l_b}{R}\right)^{1/3} \leq \phi_m .$$

Bunch length is less than difference between chord and arc length of the magnet

Bunch length is much greater than the critical wavelength of the synchrotron radiation

- We obtain a total energy loss of [1]:

$$\Delta \mathscr{E}_{tot} \simeq - \left(\frac{3^{2/3} e^2 N^2}{l_b^{4/3} R^{2/3}}\right) (R\phi_m)$$

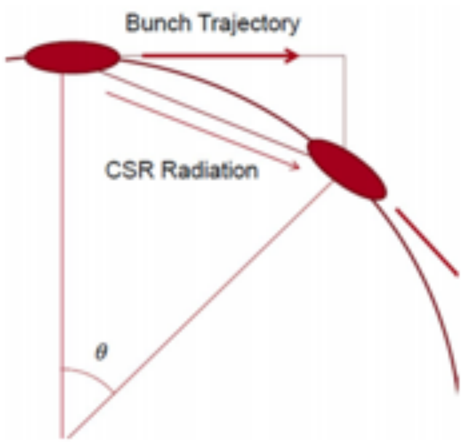
Coherent Synchrotron Radiation

$$\Delta\mathscr{E}_{\text{tot}} \simeq -\left(\frac{3^{2/3}e^2N^2}{l_{\text{b}}^{4/3}R^{2/3}}\right)(R\phi_{\text{m}})$$

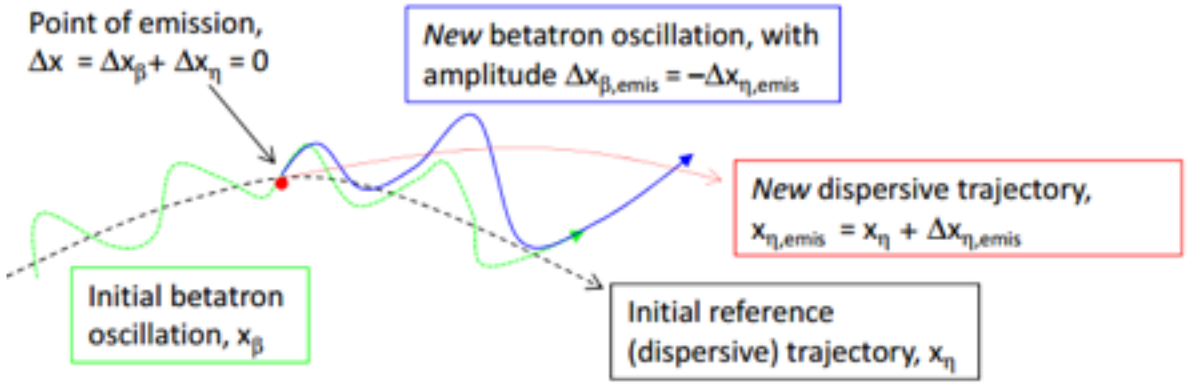
- So CSR effects will generally scale as:
  - q^2
  - N^2: It's the coherent radiation which contributes the most to this process
  - $\phi_m R^{1/3}$: Longer magnets with large bending radii will increase CSR effects
  - ${l_b}^{-4/3}$: Shorter bunches will lose more energy
- The last two points are unfortunate for bunch compressor chicanes, which need large angles (ϕ_m) to obtain a suitable change in path length so as to produce ultrashort bunches…

CSR longitudinal model

- CSR is emitted for wavelengths longer than the length of the electron bunch and leads to a detrimental tail-head interaction in bends
  - ➢The particles in the head of the bunch get energy from the back particles, so the energy dispersion gets worse. CSR longitudinal effect: $p_z(s) \rightarrow p_z(s) - \delta p_z(s)$



Picture by: SSSEPB 2015 linac and bunch compressor lecture #2, Tor Raubenheimer



Picture by: S. Di Mitri- USPAS CO-lecture_We_11

CSR effect on emittance

- $\delta p_{z,CSR}$ is correlated with z along the bunch
  - ➢All particles at the same z(slice) feel same CSR kick
  - ➢Shorter the bunch length, more intense the radiation
- The project emittance is increased by the slices misalignment in the phase space (bending plane only). Use the beam matrix to compute the emittance growth due to CSR.

$\langle x_\beta^2 \rangle$ → ; $\langle \Delta x_\beta^2 \rangle = \langle \Delta x_\eta^2 \rangle$ →

$$\varepsilon \equiv \left[\det \begin{pmatrix} \varepsilon_0 \beta + \eta^2 \sigma_{\delta,CSR}^2 & -\varepsilon_0 \alpha + \eta \eta' \sigma_{\delta,CSR}^2 \\ -\varepsilon_0 \alpha + \eta \eta' \sigma_{\delta,CSR}^2 & \varepsilon_0 \dfrac{1+\alpha^2}{\beta} + \eta'^2 \sigma_{\delta,CSR}^2 \end{pmatrix} \right]^{1/2} = \varepsilon_0 \sqrt{1 + \frac{H}{\varepsilon_0} \sigma_{\delta,CSR}^2},$$

Takes care of the coupled betatron and dispersive motion

$$\sigma_{\delta,CSR}^2 = \left\langle \left(\frac{\Delta E}{E} \right)^2 \right\rangle$$

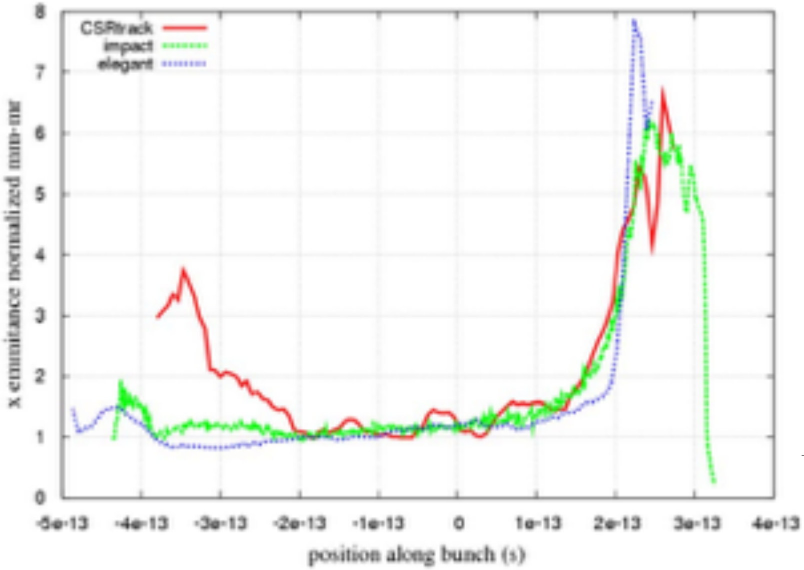
$\propto 1/\sigma_z^\tau$ where $\tau \geq 1$.

Is the CSR-induced energy spread relative to the reference particle energy

Slice emittance growth in a 4-dipoles compressor

- Slice emittance might be affected by CSR if the bunch becomes so short that particle cross over large portions of the bunch and, at the end of compression, lie in a slice different from the initial one (“phase mixing”) ⇒ incoherent “sum” of C-S invariants.

- This effect is more subtle than projected emittance growth and it is usually investigated with particle tracking codes, with a large number of particles and control of numerical noise.



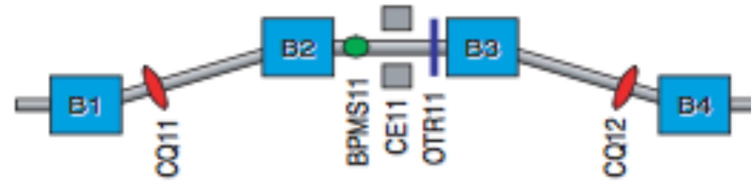
Slice emittance of a 0.8 nC, 1 mm long beam compressed by a factor 10 in BC1. The result fromelegant(sparse dotted line, with the highest peak at the right side) agrees well with that from IMPACT (thick dotted line, with sharp edge at the right side), while CSRTrack3D (solid line) predicts some emittance bumps in the core of a shorter bunch.

Cite:Di Mitri, S., et al. "Design and simulation challenges for FERMI@ elettra.NIM 608.1 (2009): 19-27.

• Techniques to mitigate CSR effects

Use tweaker quads to mitigate the CSR effects.

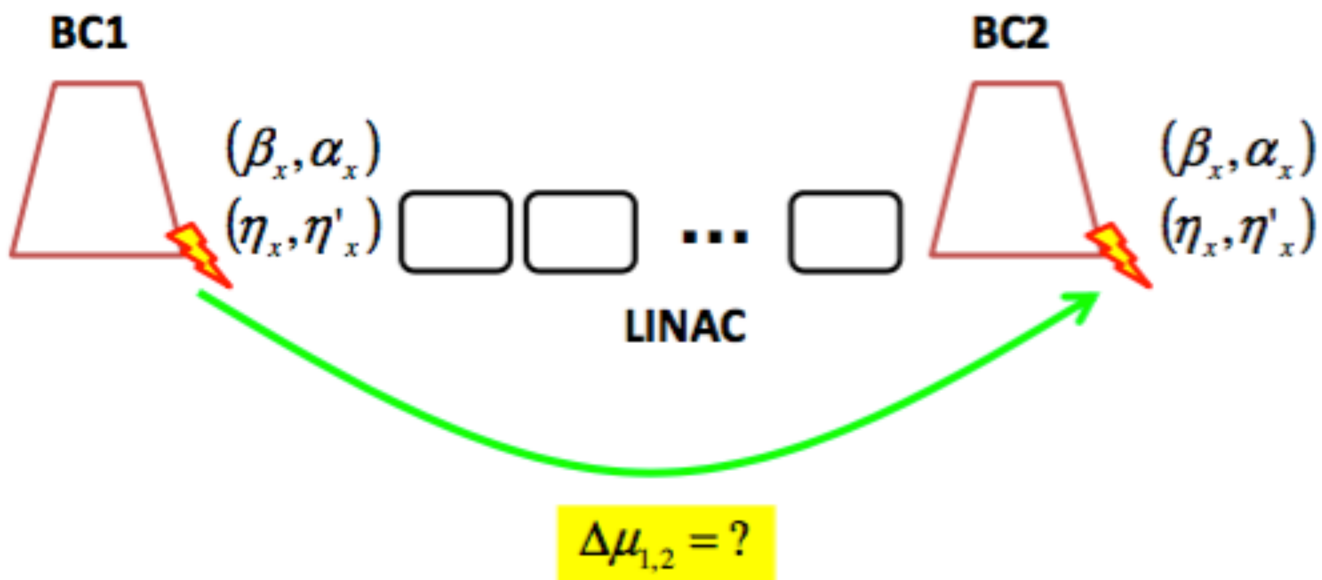
Since CSR introduces a relation between x and energy dispersion, that is, the dispersion of the whole compressor is not closed, so we try to find a way to cancel the dispersion.



Quadrupole can manipulate the final dispersion of the system (relate x/x' and energy), and the initial chirp gives a relation between energy and time. so we can use two quadrupoles to relate the x/x' and time manually, and cancel some CSR head/tail kicks.

- Techniques to mitigate CSR effects

A phase shift of pi in betatron oscillation is achieved by certain optical equipments.



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