S^{3}EPB
SLAC Summer School
on Electron and Photon Beams

Basic concepts in electron and photon beams

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SLAC and Stanford University

July 22, 2013

Lecture Outline

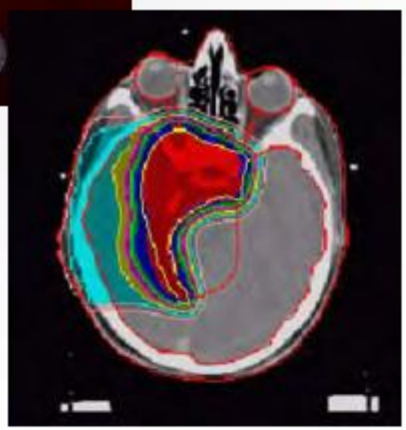
- **Electron beams**

- **Photon beams**

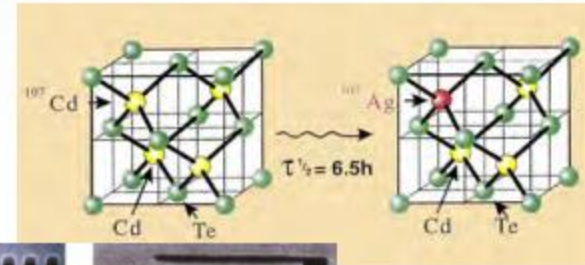
References:

1. J. D. Jackson, Classical Electrodynamics (Wiley, New York, third edition, 1999).
2. Helmut Wiedemann, Particle Accelerator Physics (Springer-Verlag, 2003).
3. *Andrew Sessler and Edmund Wilson,* Engine of Discovery (World Scientific, 2007).
4. *David Attwood, Soft X-rays and Extreme Ultraviolet Radiation (Cambridge, 1999)*
5. *Peter Schmüser, Martin Dohlus, Jörg Rossbach,* Ultraviolet and Soft X-Ray Free-Electron Lasers (Springer-Verlag, 2008).
6. Kwang-Je Kim, Zhirong Huang, Ryan Lindberg, Synchrotron Radiation and Free-Electron Lasers for Bright X-ray Sources, USPAS lecture notes 2013.
7. Gennady Stupakov, Classical Mechanics and Electromagnetism in Accelerator Physics, USPAS Lecture notes 2011.
8. Images from various sources and web sites.

Motivations:
Why does anyone care about accelerators?

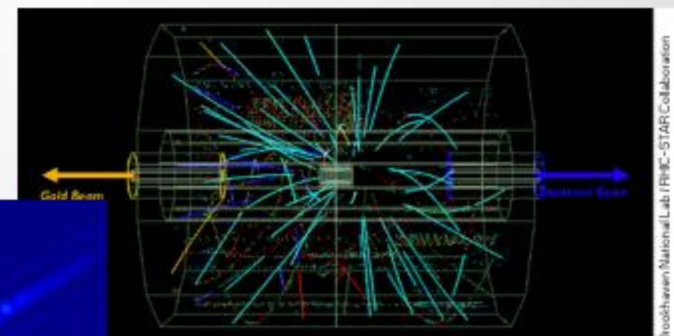


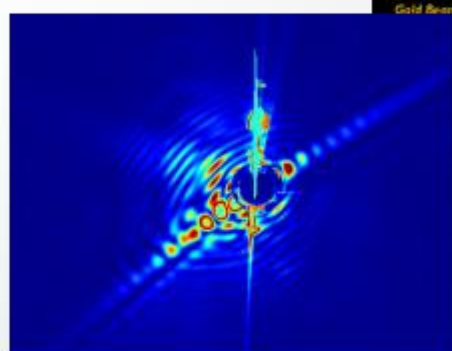
Medicine



Materials

Basic Research





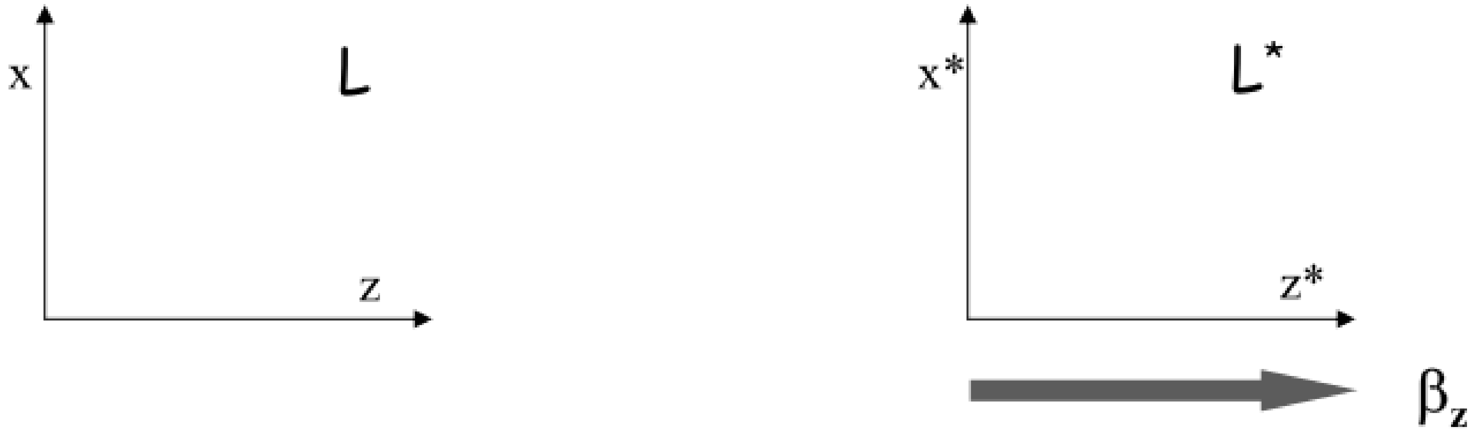
Exciting products…
exciting opportunities

William Barletta, USPAS director

Electron beams

- Primer on special relativity and E&M
- Accelerating electrons
- Transporting electrons
- Beam emittance and optics
- Beam distribution function

Lorentz Transformation



laboratory system　　　　moving system

$$\begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & \beta\gamma \\ 0 & 0 & \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} x^* \\ y^* \\ z^* \\ ct^* \end{pmatrix}$$

$$\beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

Length Contraction and Time Dilation

- Length contraction: an object of length $\Delta z'$ aligned in the moving frame with the z' axis will have the length Δz in the lab frame

$$\boxed{\Delta z = \frac{\Delta z'}{\gamma}}$$

- Time dilation: Two events occuring in the moving frame at the same point and separated by the time interval $\Delta t'$ will be measured by the lab observers as separated by Δt

$$\boxed{\Delta t = \gamma \Delta t'}$$

Energy, Mass, Momentum

- Energy

$$\boxed{E = T + mc^2}$$

Kinetic energy   Rest mass energy

- Electrons rest mass energy 511 keV (938 MeV for protons), 1eV = 1.6 × 10^{-19} Joule

- Momentum

$$\boxed{\boldsymbol{p} = \gamma \boldsymbol{\beta} mc}$$

- Energy and momentum

$$\boxed{\begin{array}{l} E^2 = p^2c^2 + m^2c^4, \\ E = \gamma mc^2. \end{array}}$$

Maxwell's Equations

$$\nabla \cdot \boldsymbol{D} = \rho$$
$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$
$$\nabla \times \boldsymbol{H} = \boldsymbol{j} + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E}$$
$$\boldsymbol{B} = \mu_0 \boldsymbol{H}$$

$$c = (\epsilon_0 \mu_0)^{-1/2}$$

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 377\ \mathrm{Ohm}$$

- Wave equation

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \nabla^2\right) \boldsymbol{E} = -\frac{1}{\epsilon_0}\left(\frac{\partial \boldsymbol{j}}{\partial t} + c^2 \nabla \rho\right)$$

- Lorentz transformation of fields

$$E_z = E'_z\,, \qquad \boldsymbol{E}_\perp = \gamma\left(\boldsymbol{E}'_\perp - \boldsymbol{v} \times \boldsymbol{B}'\right)\,,$$
$$B_z = B'_z\,, \qquad \boldsymbol{B}_\perp = \gamma\left(\boldsymbol{B}'_\perp + \frac{1}{c^2}\boldsymbol{v} \times \boldsymbol{E}'\right)$$

Lorentz Force

- Lorentz force

$$\boxed{\boldsymbol{F} = e\boldsymbol{E} + e\boldsymbol{v} \times \boldsymbol{B}}$$

- Momentum and energy change

$$\Delta \boldsymbol{p} = \int \boldsymbol{F} dt$$

$$\Delta E = \int \boldsymbol{F} d\boldsymbol{s}$$

$$\boxed{d\boldsymbol{s} = \boldsymbol{v} dt}$$

- **Energy exchange through *E* field only**

$$\Delta E = \int \boldsymbol{F} d\boldsymbol{s} = e \int \boldsymbol{E} \cdot d\boldsymbol{s} + e \int \underbrace{(\boldsymbol{v} \times \boldsymbol{B}) \cdot \boldsymbol{v}}_{= 0} dt$$

No work done by magnetic field!

A relativistic electron

$$\gamma = \frac{E_e}{mc^2} = \frac{E_e[\mathrm{GeV}]}{0.511 \times 10^{-3}} = 1957\, E_e[\mathrm{GeV}]$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} \approx 1 - \frac{1}{2\gamma^2},$$

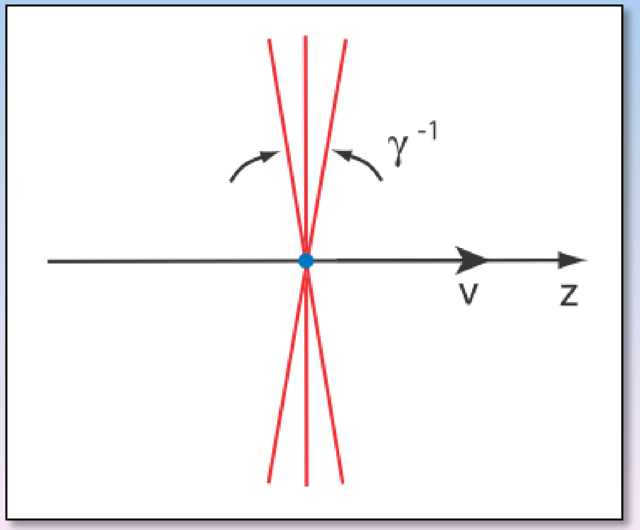
$$1 - \beta \approx 5 \times 10^{-8} \quad \text{for} \quad E_e = 1.5 \text{ GeV}$$

- In electron's frame, Coulumb field is

$$\boldsymbol{E}' = \frac{1}{4\pi\epsilon_0} \frac{e\boldsymbol{r}'}{r'^3}$$

- In lab frame, space charge field is

$$E_x = \frac{1}{4\pi\epsilon_0} \frac{e\gamma x}{[x^2 + y^2 + \gamma^2(z - vt)^2]^{3/2}}$$
$$E_y = \frac{1}{4\pi\epsilon_0} \frac{e\gamma y}{[x^2 + y^2 + \gamma^2(z - vt)^2]^{3/2}}$$
$$E_z = \frac{1}{4\pi\epsilon_0} \frac{e\gamma(z - vt)}{[x^2 + y^2 + \gamma^2(z - vt)^2]^{3/2}}$$



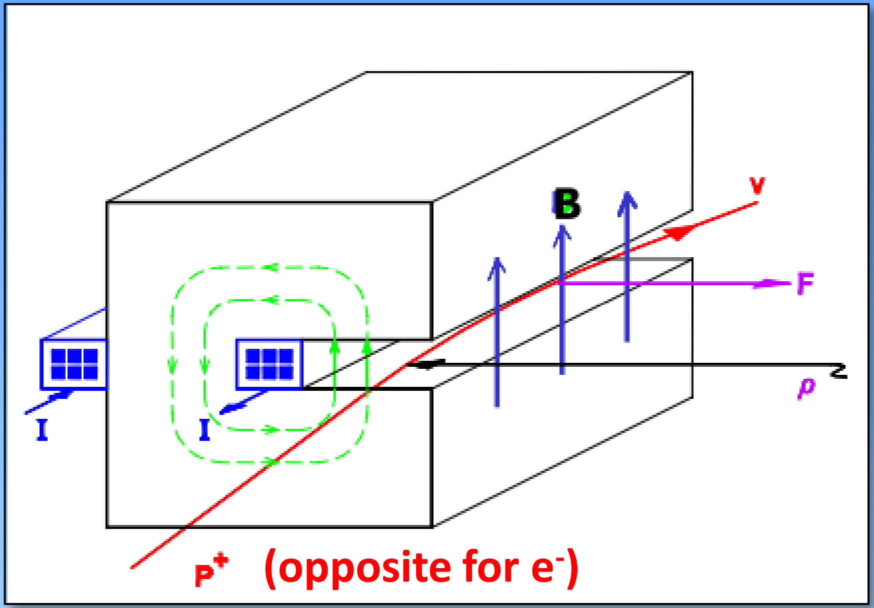
Guiding beam: dipole

- Lorentz force

$$\boldsymbol{F} = \cancel{e\boldsymbol{E}} + e\boldsymbol{v} \times \boldsymbol{B}$$

- Centrifugal force

$$F_{cf} = \frac{\gamma m c^2 \beta^2}{\rho}$$



- Bending radius is obtained by balance the forces

$$\frac{1}{\rho} = \frac{eB}{\gamma \beta m c^2}$$

$$\frac{1}{\rho}\,[\mathrm{m}^{-1}] = 0.2998 \frac{B[\mathrm{T}]}{\beta E[\mathrm{GeV}]}$$

Cyclotron

- If beam moves circularly, re-traverses the same accelerating section again and again, we can accelerate the beam repetitively

Ernest O. Lawrence in 1930

The first cyclotron with a diameter of 5 inches

[Ref.]: Photography gallery of Lawrence Berkeley National Laboratory, http://cso.lbl.gov/photo/gallery/

Lawrence started to construct a cyclotron, as the machine later was named, in early 1930. A graduate student, M. Stanley Livingston, did much of the work of translating the idea into working hardware. In January 1931 Lawrence and Livingston met their first success. A device about 4.5 inches in diameter used a potential of 1,800 volts to accelerate hydrogen ions up to energies of 80,000 electron volts. Lawrence immediately started planning for a bigger machine. In summer 1931 an eleven-inch cyclotron achieved a million volts.

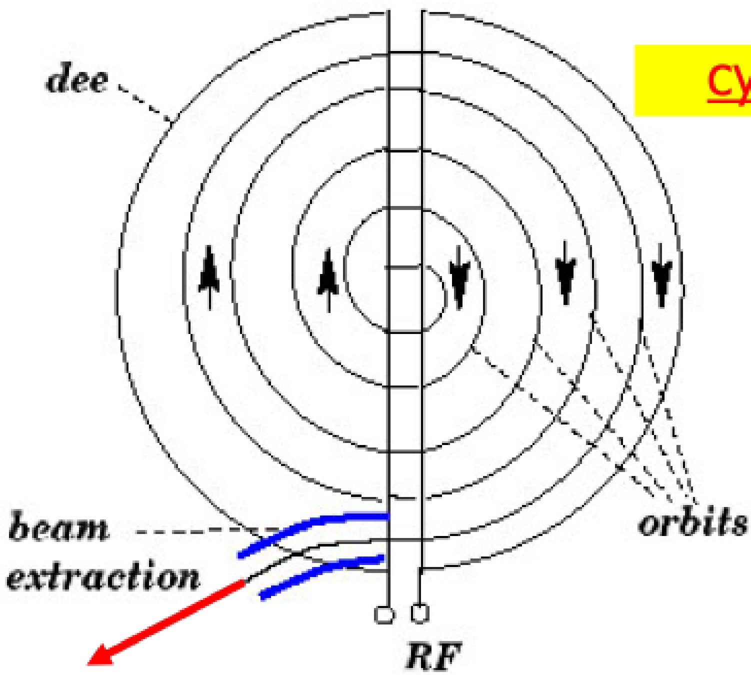
"Dr Livingston has asked me to advise you that he has obtained 1,100,000 volt protons. He also suggested that I add 'Whoopee'!"

—Telegram to Lawrence,
3 August 1931

http://www.aip.org/history/lawrence/first.htm

From Cyclotron to Synchrotron

- Cyclotron does not work for relativistic beams.



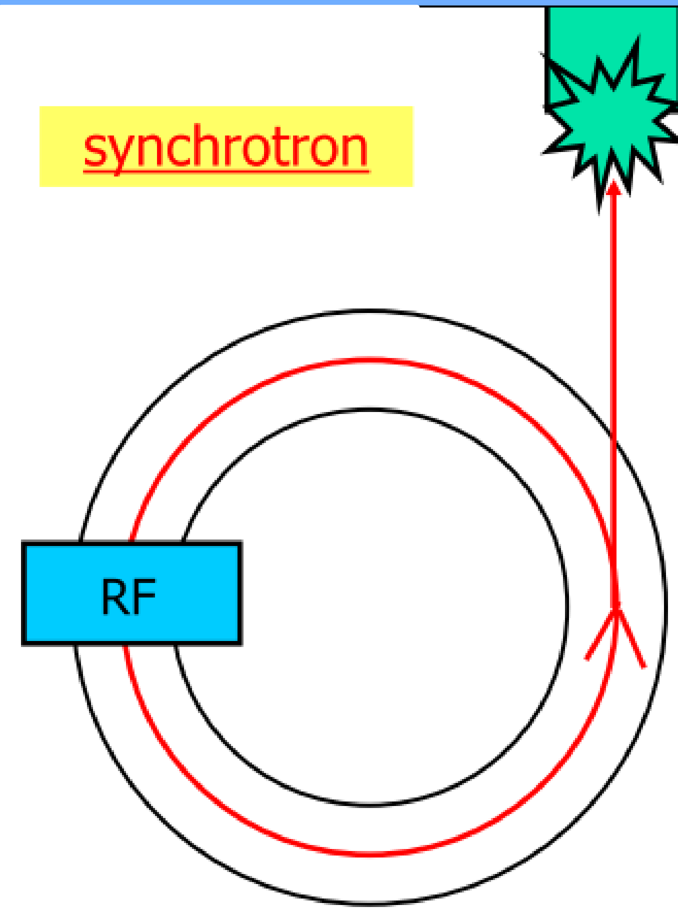
cyclotron

huge dipole, compact design,

B = constant

low energy, single pass.

synchrotron

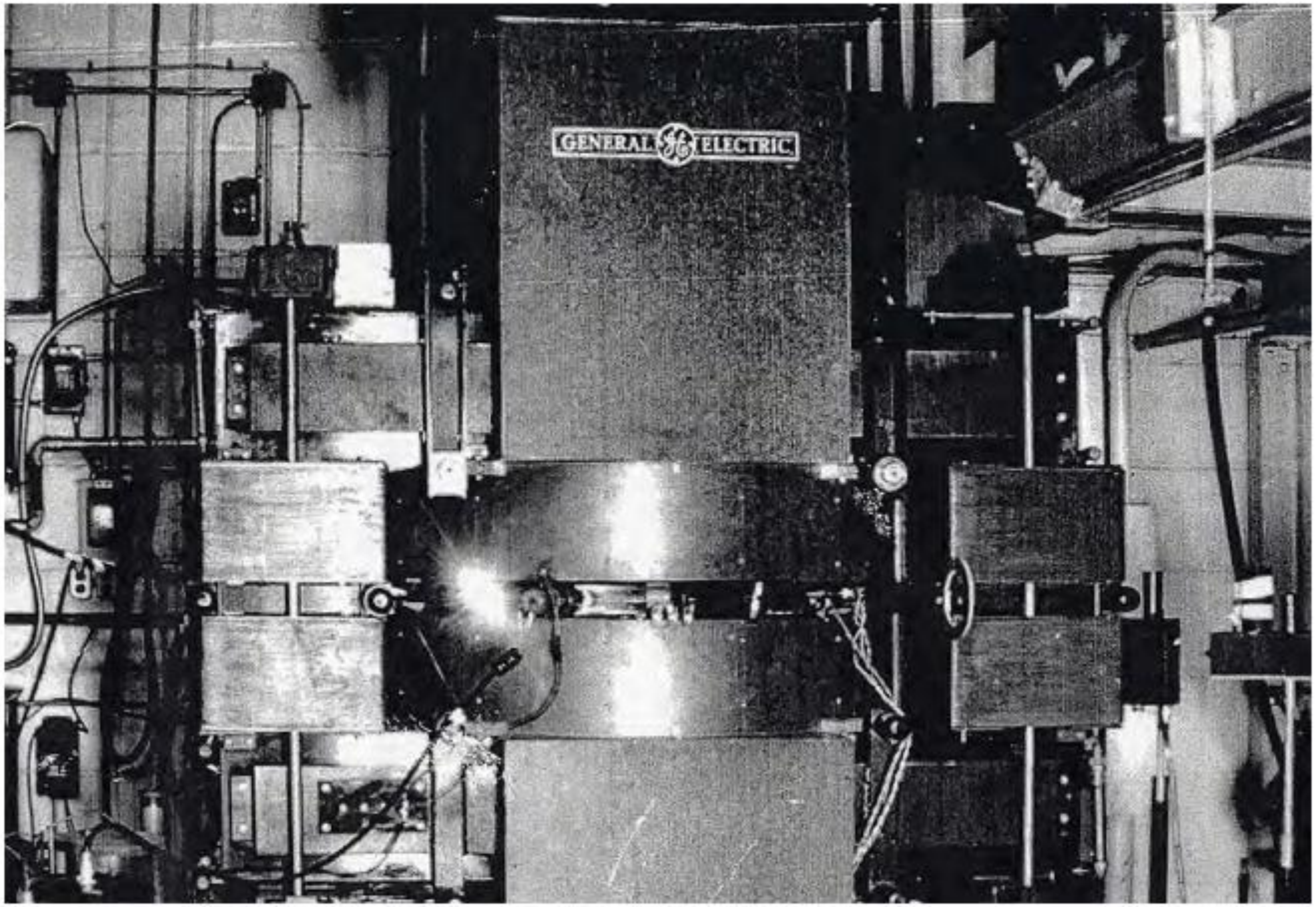


varying B, small magnets, high energy

Synchrotron

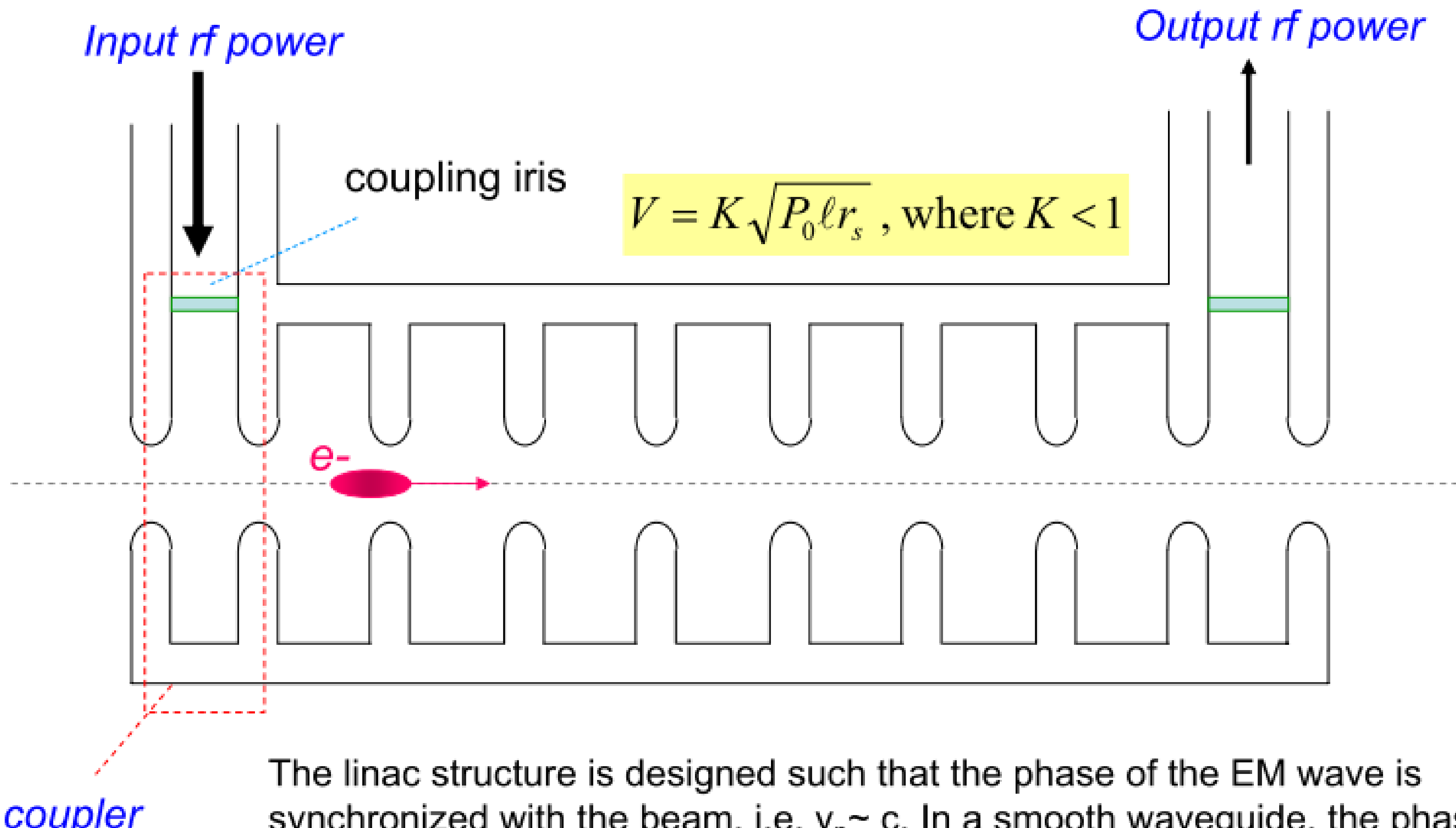
- GE synchrotron observed first synchrotron radiation (1946) and opened a new era of accelerator-based light sources.

The first purpose-built synchrotron to operate was built with a glass vacuum chamber



Electron linac

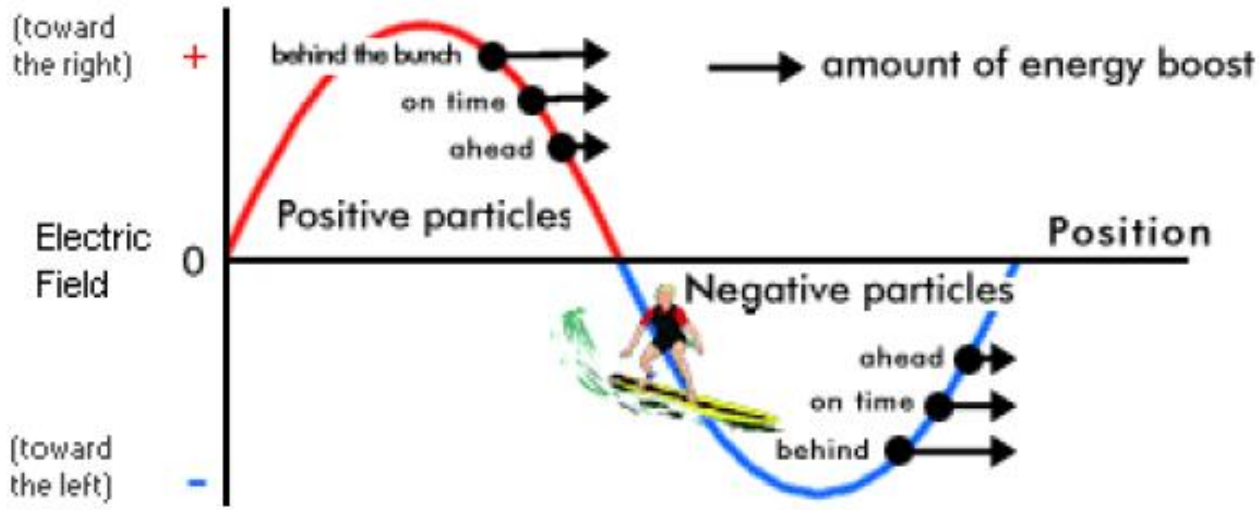
The rf energy is used to launch a traveling wave or standing wave in an array of cavities.

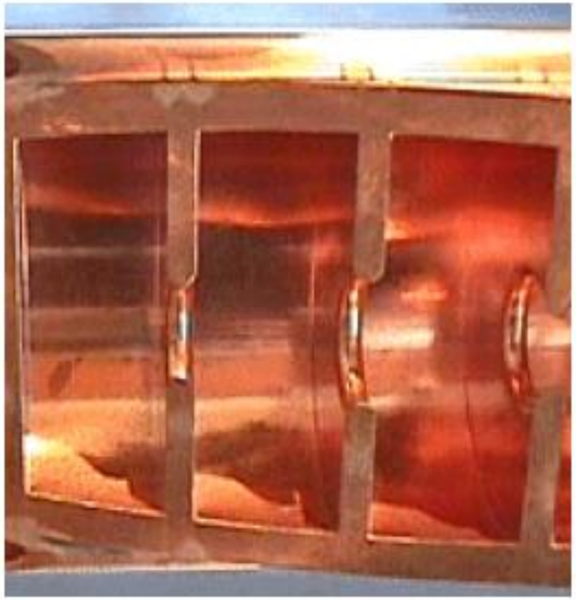


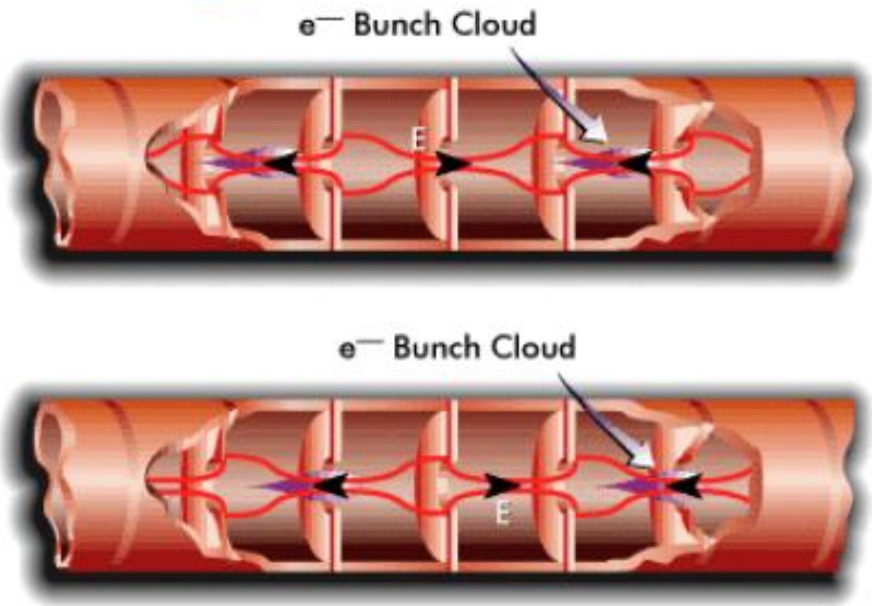
$$V = K\sqrt{P_0 \ell r_s}\ \text{, where } K < 1$$

The linac structure is designed such that the phase of the EM wave is synchronized with the beam, i.e. $v_p \sim c$. In a smooth waveguide, the phase velocity $v_p > c$. Those disks are used to slow down the waveguide phase velocity in order to achieve synchronism with the electron beams.

Electron Linac (disk loaded structure)



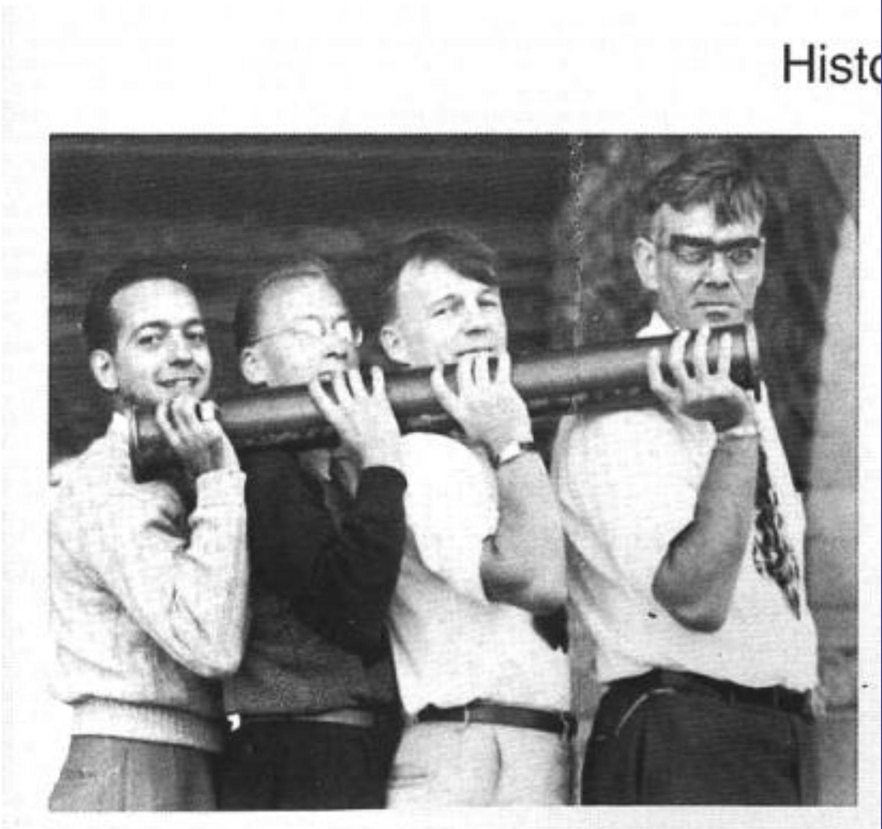


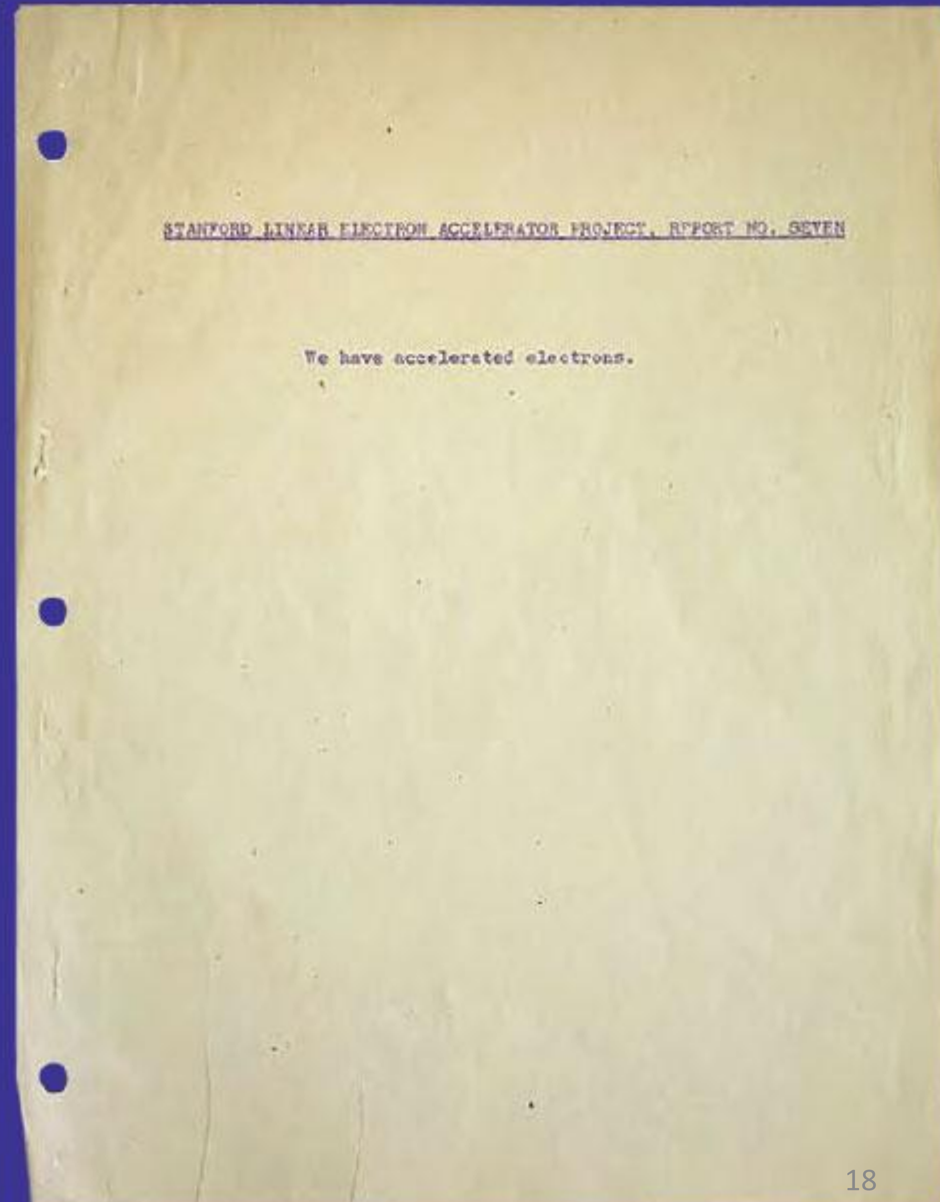


1/20,000,000,000 second later
(notice how far the bunches have moved)

[Ref.] http://www.slac.stanford.edu

Disk loaded structure made at Stanford Univ. (1947)

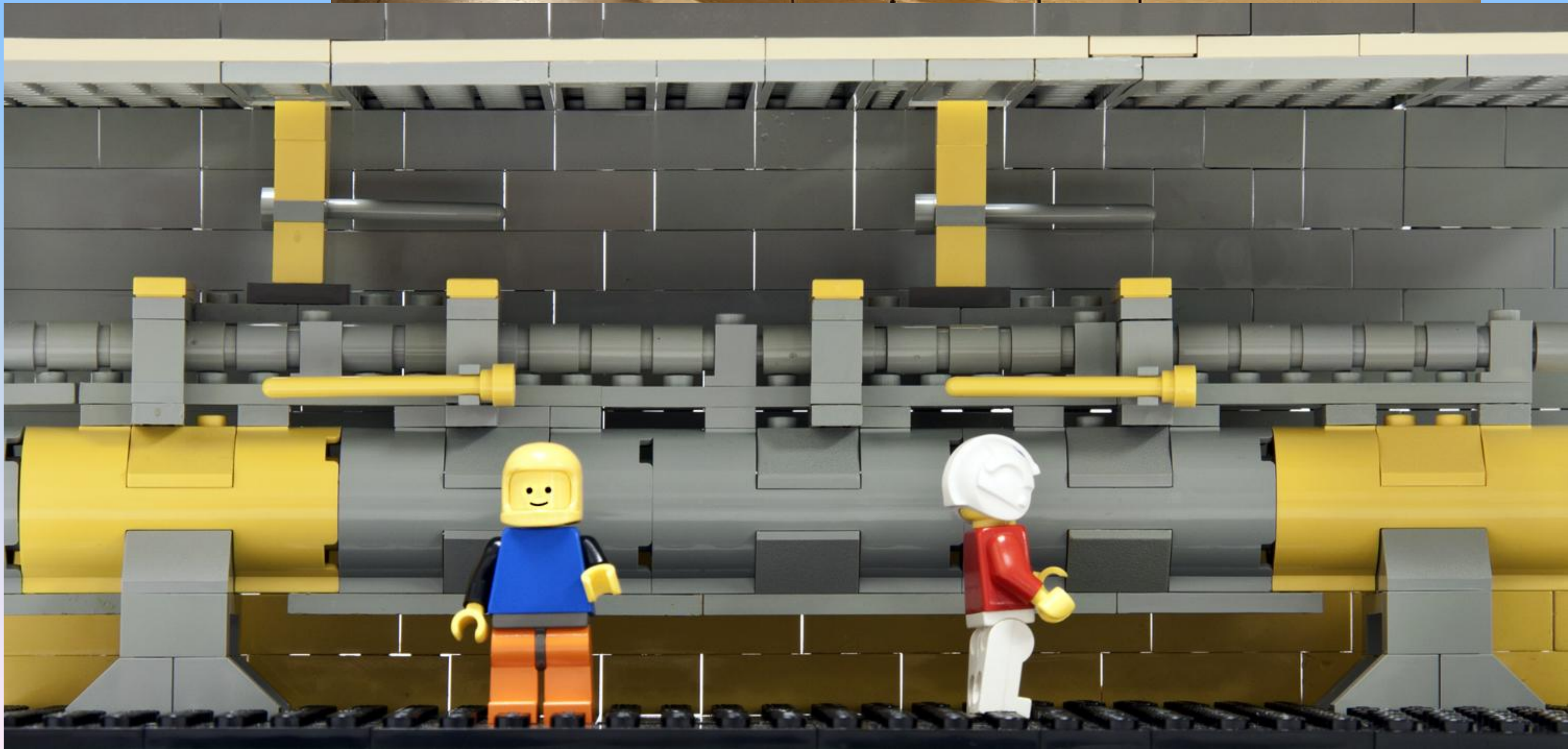




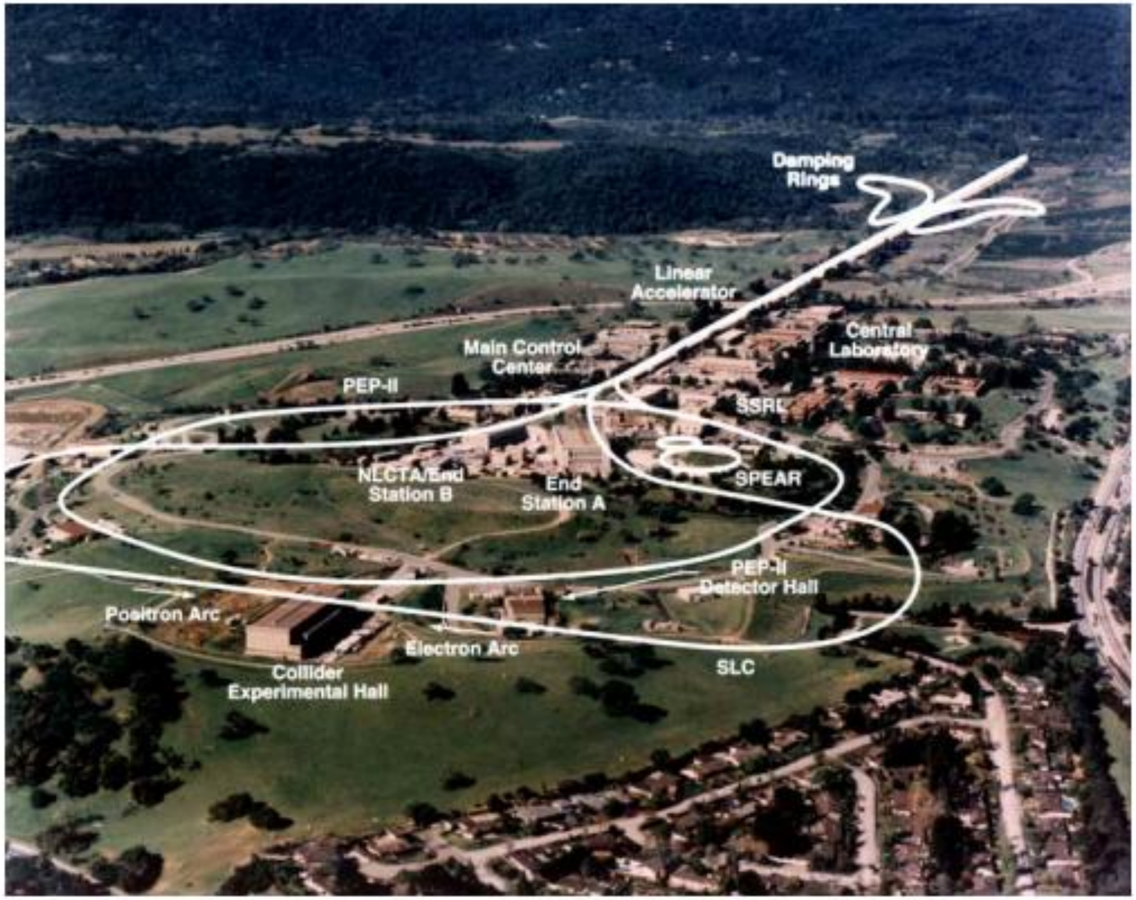
STANFORD LINEAR ELECTRON ACCELERATOR PROJECT, REPORT NO. SEVEN

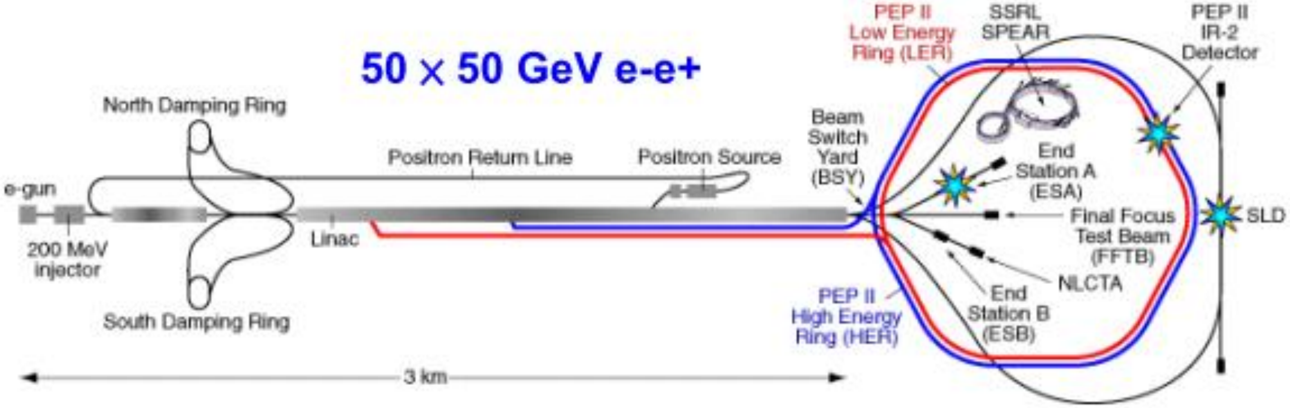
We have accelerated electrons.

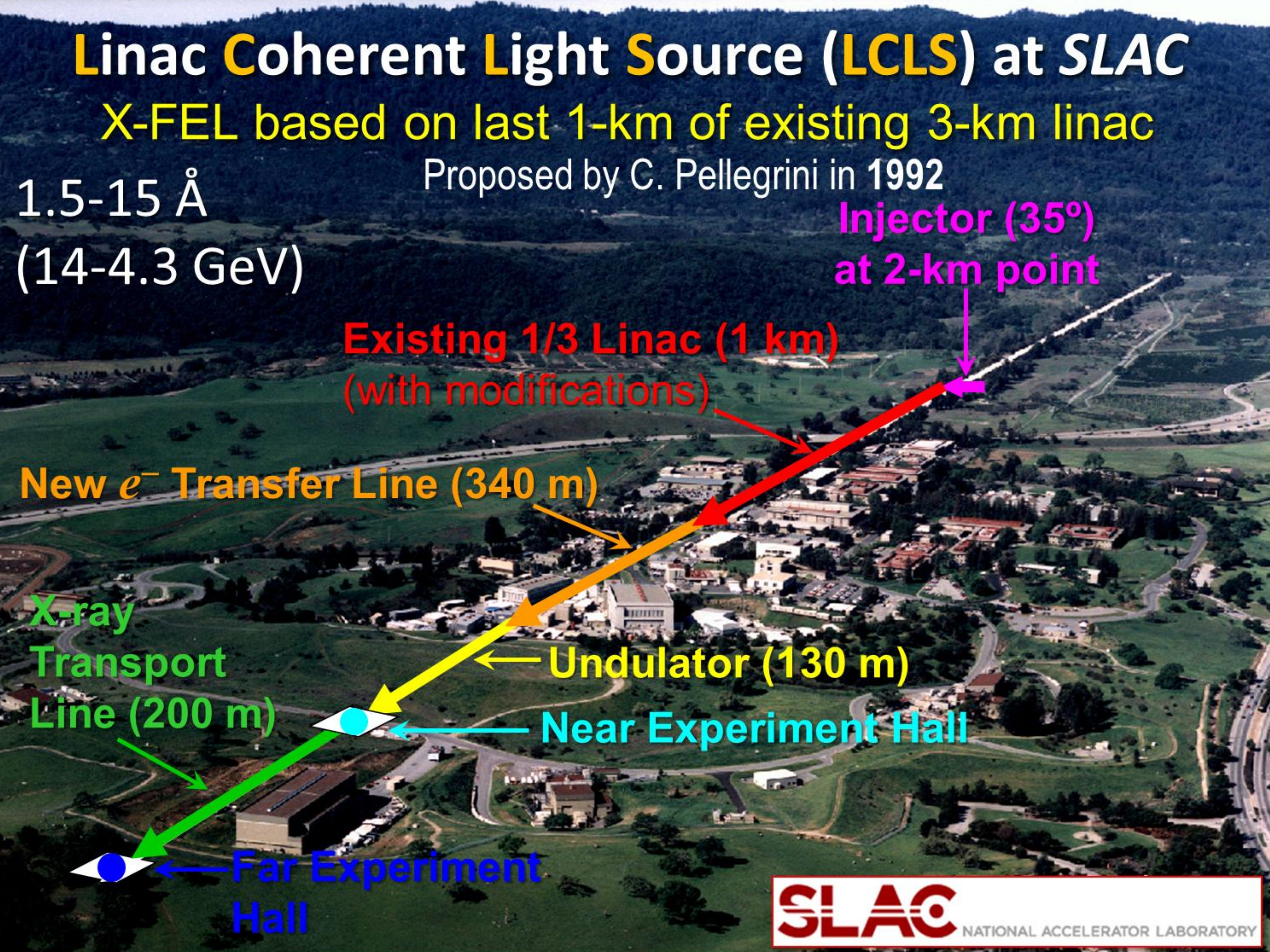
SLAC linac



Stanford Linear Accelerator Center (SLAC)

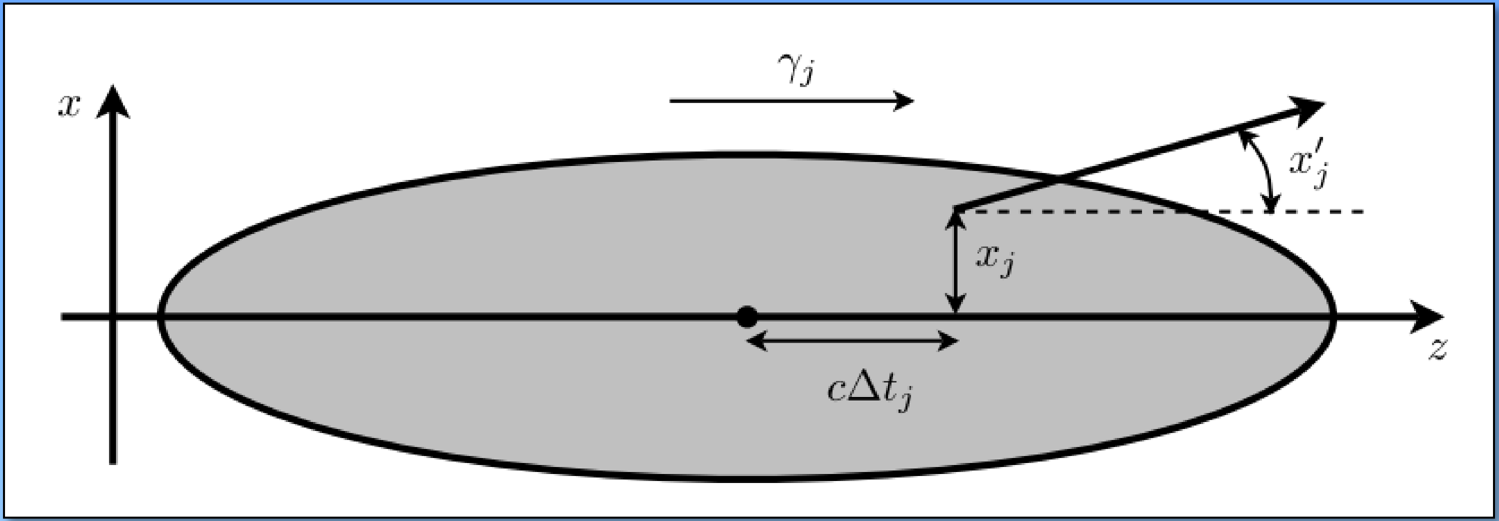






Linac Coherent Light Source (LCLS) at SLAC
X-FEL based on last 1-km of existing 3-km linac
Proposed by C. Pellegrini in 1992
1.5-15 Å
(14-4.3 GeV)
Injector (35º)
at 2-km point
Existing 1/3 Linac (1 km)
(with modifications)
New e^- Transfer Line (340 m)
X-ray
Transport
Line (200 m)
Undulator (130 m)
Near Experiment Hall
Far Experiment
Hall
SLAC NATIONAL ACCELERATOR LABORATORY

Beam description



- Beam phase space ($x, x', y, y', \Delta t, \Delta\gamma$)

$$x' \equiv \frac{dx}{dz} = \frac{dx/dt}{dz/dt} = \frac{1}{v_z}\frac{dx}{dt}$$

$$\Delta\gamma_j \equiv \gamma_j - \gamma_0$$

- Consider paraxial beams such that

$$|\boldsymbol{x}'| = \sqrt{x'^2 + y'^2} \approx \frac{1}{c}\sqrt{v_x^2 + v_y^2} \ll 1$$

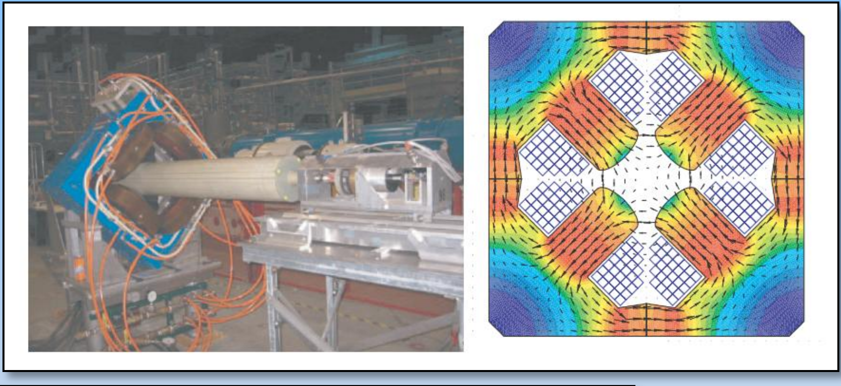
Linear beam transport

- Transport matrix

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{\mathrm{o}} = \mathsf{M}(z_{\mathrm{i}}, z_{\mathrm{o}}) \begin{bmatrix} x \\ x' \end{bmatrix}_{\mathrm{i}}$$

- Free space drift

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{\mathrm{o}} = \begin{bmatrix} 1 & \ell \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_{\mathrm{i}} \equiv \mathsf{M}_{\ell} \begin{bmatrix} x \\ x' \end{bmatrix}_{\mathrm{i}}$$



- Quadrupole (de-)focusing

$$\begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_{\mathrm{o}} = \mathsf{M}_f \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_{\mathrm{i}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/f & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/f & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}_{\mathrm{i}}$$

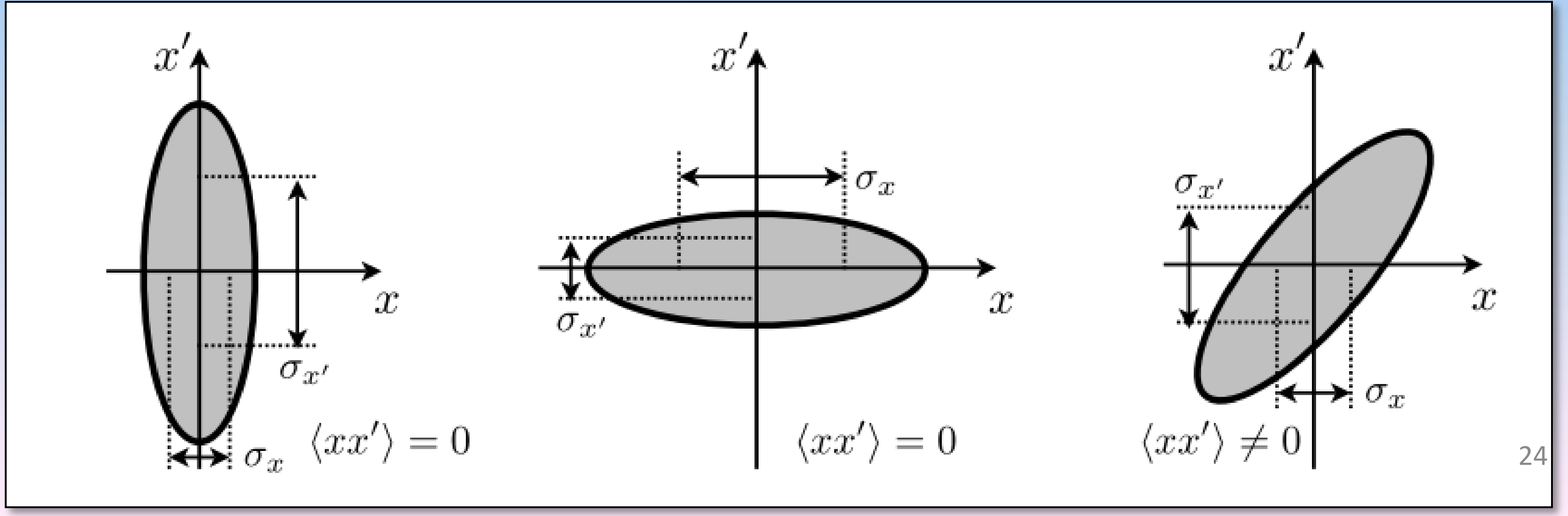
Beam properties

- Second moments of beam distribution

rms size
$$\sigma_x^2(z) = \langle x^2 \rangle = \frac{1}{N_e}\sum_j x_j^2.$$

rms divergence
$$\sigma_{x'}^2(z) = \left\langle x'^2 \right\rangle = \frac{1}{N_e}\sum_j x_j'^2.$$

correlation
$$\langle x\, x' \rangle = \frac{1}{N_e}\sum_j x_j x_j'.$$



Beam emittance

- **Emittance or geometric emittance**

$$\varepsilon_x \equiv \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle x x' \rangle}$$

- Emittance is **conserved** in a **linear** transport system

- **Normalized emittance** is conserved in a linear system including acceleration

$$\varepsilon_{x,n} = \beta_z \gamma \varepsilon_x \approx \gamma \varepsilon_x$$

- Normalized emittance is hence an important figure of merit for electron sources

- Preservation of emittances is critical for accelerator designs.

Beam optics function

- Optics functions (Twiss parameters)

$$\beta_x = \frac{\langle x^2 \rangle}{\varepsilon_x} \qquad \gamma_x = \frac{\langle x'^2 \rangle}{\varepsilon_x} \qquad \alpha_x = -\frac{\langle x x' \rangle}{\varepsilon_x}$$

$$\beta_x \gamma_x - \alpha_x^2 = 1$$

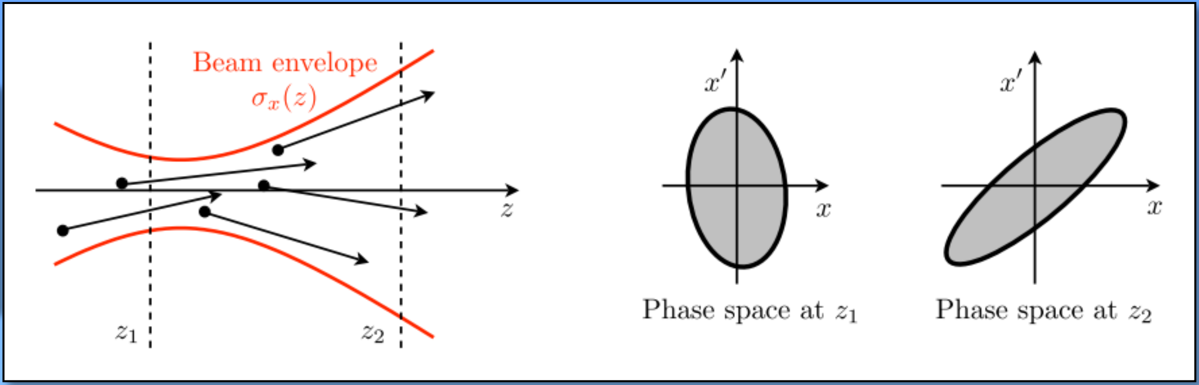
- Given beta function along beamline

$$\sigma_x(z) = \sqrt{\varepsilon_x \beta_x(z)}$$

Free space propagation

- Single particle

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{\mathrm{o}} = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_{\mathrm{i}}$$



- Beam envelope

$$\langle x_{\mathrm{o}}^2 \rangle = \left\langle \left(x_{\mathrm{i}} + z\, x'_{\mathrm{i}} \right)^2 \right\rangle = \langle x_{\mathrm{i}}^2 \rangle + 2z \,\langle x_{\mathrm{i}} x'_{\mathrm{i}} \rangle + z^2 \langle x'^2_{\mathrm{i}} \rangle$$

$$\beta_x(z) = \beta_x(0) + z^2 \gamma_x(0)$$

$$\beta_x(z) = \beta_x^* + \frac{z^2}{\beta_x^*}$$

$$\sigma_x(z) = \sqrt{\varepsilon_x \beta_x(z)} = \sqrt{\varepsilon_x \left(\beta_x^* + \frac{z^2}{\beta_x^*} \right)}.$$

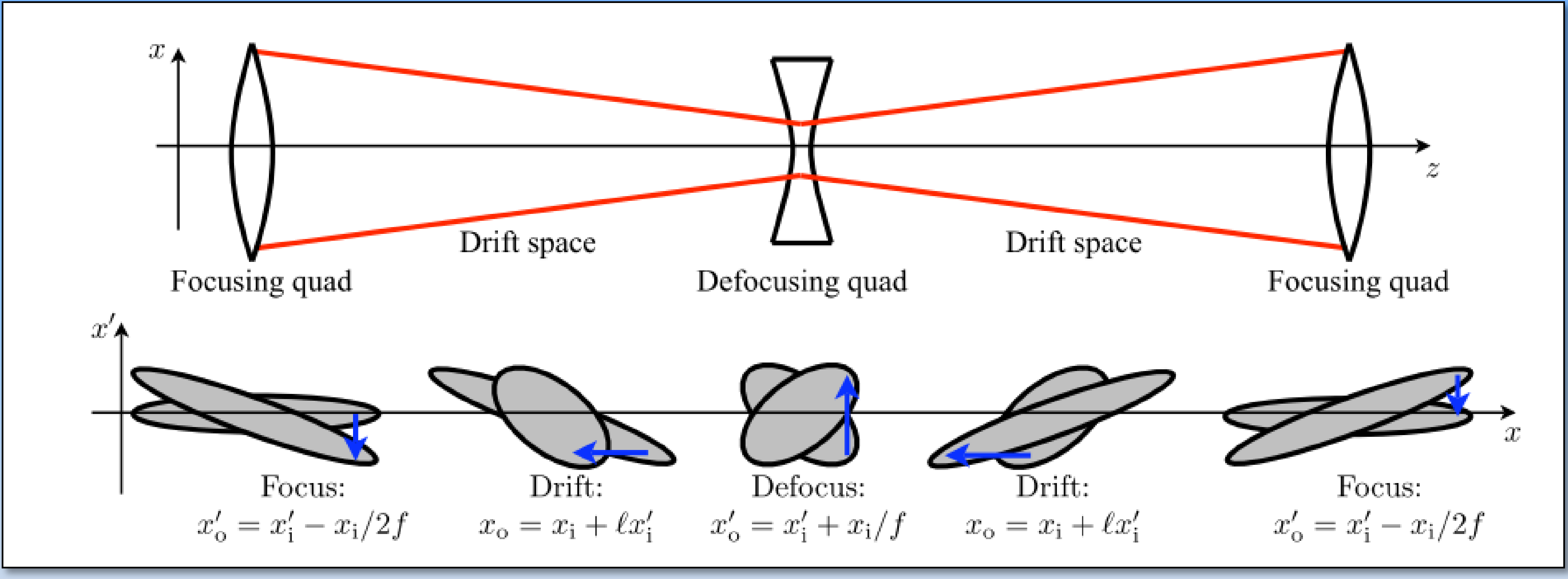
- Analogous with Gaussian laser beam

$$\varepsilon_x \leftrightarrow \frac{\lambda}{4\pi} \qquad \beta_x^* \leftrightarrow Z_R.$$

FODO lattice

- Multiple elements (e. g., FODO lattice)

$$\mathsf{M} = \mathsf{M}_N \, \mathsf{M}_{N-1} \, \ldots \, \mathsf{M}_2 \, \mathsf{M}_1$$



$$\begin{aligned}
\mathsf{M}_{\mathrm{FODO}} &= \begin{bmatrix} 1 & 0 \\ -1/2f & 1 \end{bmatrix} \begin{bmatrix} 1 & \ell \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/f & 1 \end{bmatrix} \begin{bmatrix} 1 & \ell \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/2f & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1 - \frac{\ell^2}{2f^2} & 2\ell\left(1 + \frac{\ell}{2f}\right) \\ -\frac{\ell}{2f^2}\left(1 - \frac{\ell}{2f}\right) & 1 - \frac{\ell^2}{2f^2} \end{bmatrix}.
\end{aligned}$$

FODO lattice II

For periodic motion we have $\beta_x(0) = \beta_x(2\ell)$ and $\gamma_x(0) = \gamma_x(2\ell)$, while vanishing correlation α_x at the two planes implies that $\beta_x(0) = 1/\gamma_x(0)$

- Maximum beta

$$\beta_x(0) = 2\sqrt{\frac{2f^3 + f^2\ell}{2f - \ell}} \approx 2\,|f|\left(1 + \frac{\ell}{2f}\right)$$

- Minimum beta

$$\beta_x(\ell) \approx 2\,|f|\left(1 - \frac{\ell}{2f}\right)$$

- When $f >> l$

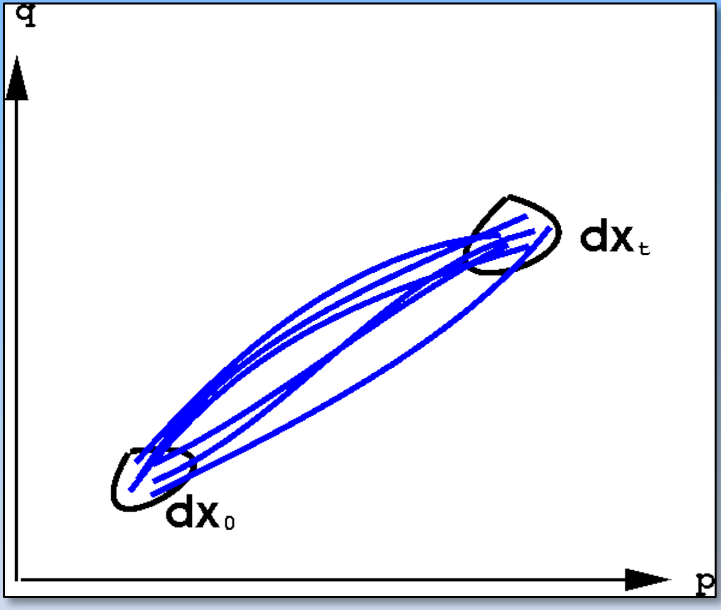
$$\begin{aligned}
\beta_x(z) \approx \bar{\beta}_x = 2f \quad & \longrightarrow \quad \langle x^2 \rangle \approx 2\varepsilon_x f \\
\gamma_x(z) \approx \frac{2}{\bar{\beta}_x} = \frac{1}{f} \quad & \longrightarrow \quad \langle x'^2 \rangle \approx \frac{\varepsilon_x}{f} \\
\alpha_x^2(z) \approx \bar{\beta}_x \bar{\gamma}_x - 1 = 1 \quad & \longrightarrow \quad \langle x x' \rangle \approx \pm\varepsilon_x .
\end{aligned}$$

Electron distribution in phase space

- We define the distribution function *F* so that

$$N_e F(\Delta t, \Delta\gamma, \boldsymbol{x}, \boldsymbol{x}'; z)\, d\boldsymbol{x} d\boldsymbol{x}' d(\Delta t) d(\Delta\gamma)$$

is the number of electrons per unit phase space volume



- Since the number of electrons is an invariant function of *z*, distribution function satisfies **Liouville theorem**

$$\frac{d}{dz}F = \left[\frac{\partial}{\partial z} + (\Delta t)'\frac{\partial}{\partial \Delta t} + (\Delta\gamma)'\frac{\partial}{\partial \Delta\gamma} + \boldsymbol{x}'\cdot\frac{\partial}{\partial \boldsymbol{x}} + \boldsymbol{x}''\cdot\frac{\partial}{\partial \boldsymbol{x}'}\right]F = 0$$

equations of motion

Gaussian beam distribution

- Represent the ensemble of electrons with a continuous distribution function (e.g., Gaussian in *x* and *x'*)

$$F(x,x';z)=\frac{1}{2\pi\varepsilon_x}\exp\left\{-\frac{1}{2\varepsilon_x}\left[\gamma_x(z)x^2+\beta_x(z)x'^2+2\alpha_x(z)xx'\right]\right\}$$

- For free space propagation

$$F(x,x';z)=\frac{1}{2\pi\varepsilon_x}\exp\left[-\frac{(x-x'z)^2}{2\beta_x^*\varepsilon_x}-\frac{x'^2}{2\varepsilon_x/\beta_x^*}\right]$$

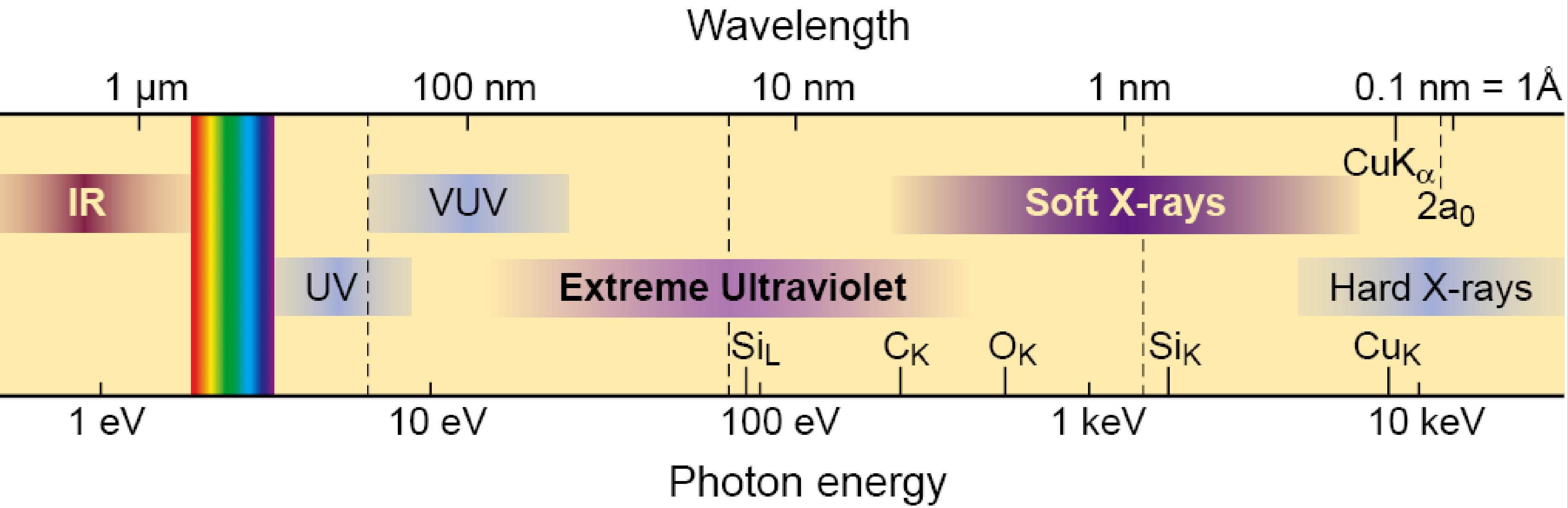
- Distribution in physical space can be obtained by integrating *F* over the angle

$$\int dx'\;F(x,x';z)=\frac{\exp\left[-\frac{x^2}{2\sigma_x^{*2}(1+z^2/\beta_x^{*2})}\right]}{\sqrt{2\pi}\,\sigma_x^*\sqrt{1+z^2/\beta_x^{*2}}}$$

Photon beams

- Wave equation and paraxial approximation
- Radiation diffraction and emittance
- Transverse and temporal coherence
- Brightness and diffraction limit
- Radiation intensity and bunching
- Bright accelerator based photon sources

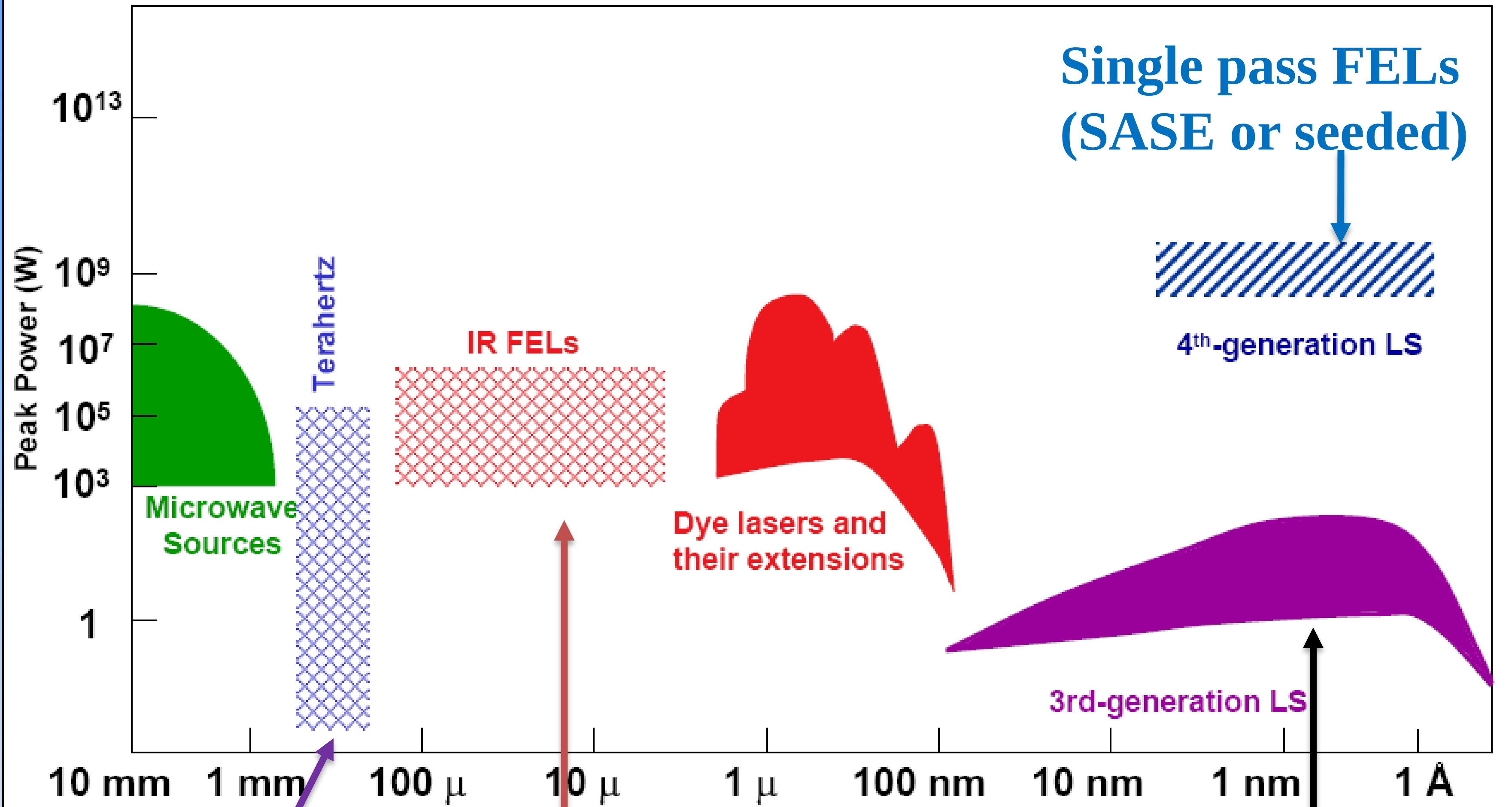
Photon wavelength and energy



- **See smaller features**
- **Write smaller patterns**
- **Elemental and chemical sensitivity**

$$\boxed{\hbar\omega \cdot \lambda = hc = 1239.842 \text{ eV nm}}$$

Opportunities for Tunable Source of Radiation



Synchrotron radiation
Undulator radiation

Various accelerator
and non-acc. sources

FEL oscillators
(High-average power)

Radiation diffraction

- Wave propagation in free space

$$\left[\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial \boldsymbol{x}^2} + k^2\right] E_\omega(\boldsymbol{x}; z) = 0, \quad k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$$

- Angular representation

$$\mathcal{E}_\omega(\boldsymbol{\phi}; z) = \frac{1}{\lambda^2}\int d\boldsymbol{x}\, e^{-ik\boldsymbol{\phi}\cdot\boldsymbol{x}} E_\omega(\boldsymbol{x}; z)$$

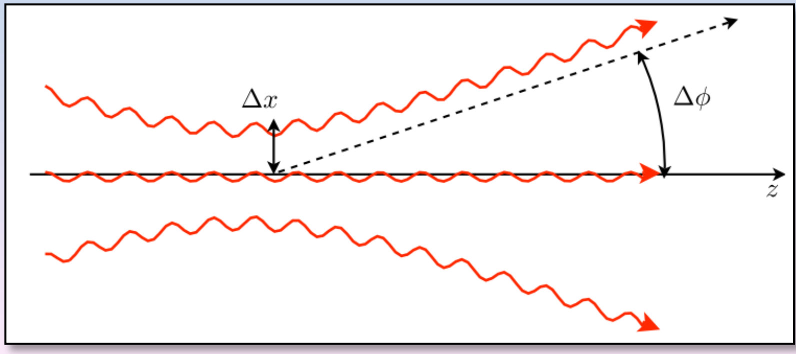
$$E_\omega(\boldsymbol{x}; z) = \int d\boldsymbol{\phi}\, e^{ik\boldsymbol{\phi}\cdot\boldsymbol{x}} \mathcal{E}_\omega(\boldsymbol{\phi}; z).$$

- General solution

$$E_\omega(\boldsymbol{x}; z) = \int d\boldsymbol{\phi}\ \exp\left[ik(\boldsymbol{\phi}\cdot\boldsymbol{x} \pm z\sqrt{1-\phi^2})\right] \mathcal{E}_\omega(\boldsymbol{\phi}; 0)$$

- Paraxial approximation (ϕ^2 << 1)

$$\mathcal{E}_\omega(\boldsymbol{\phi}; z) = e^{ik(1-\phi^2/2)z} \mathcal{E}_\omega(\boldsymbol{\phi}; 0)$$



Gaussian beam and radiation emittance

- Gaussian fundamental mode at waist *z=0*

$$E(x;0) = E_0 \exp\left(-\frac{x^2}{4\sigma_r^2}\right)$$

$$\mathcal{E}(\phi;0) = \mathcal{E}_0 \exp\left(-\frac{\phi^2}{4\sigma_{r'}^2}\right)$$

$$\sigma_r \sigma_{r'} = \frac{\lambda}{4\pi} \equiv \varepsilon_r$$

- At arbitrary *z*

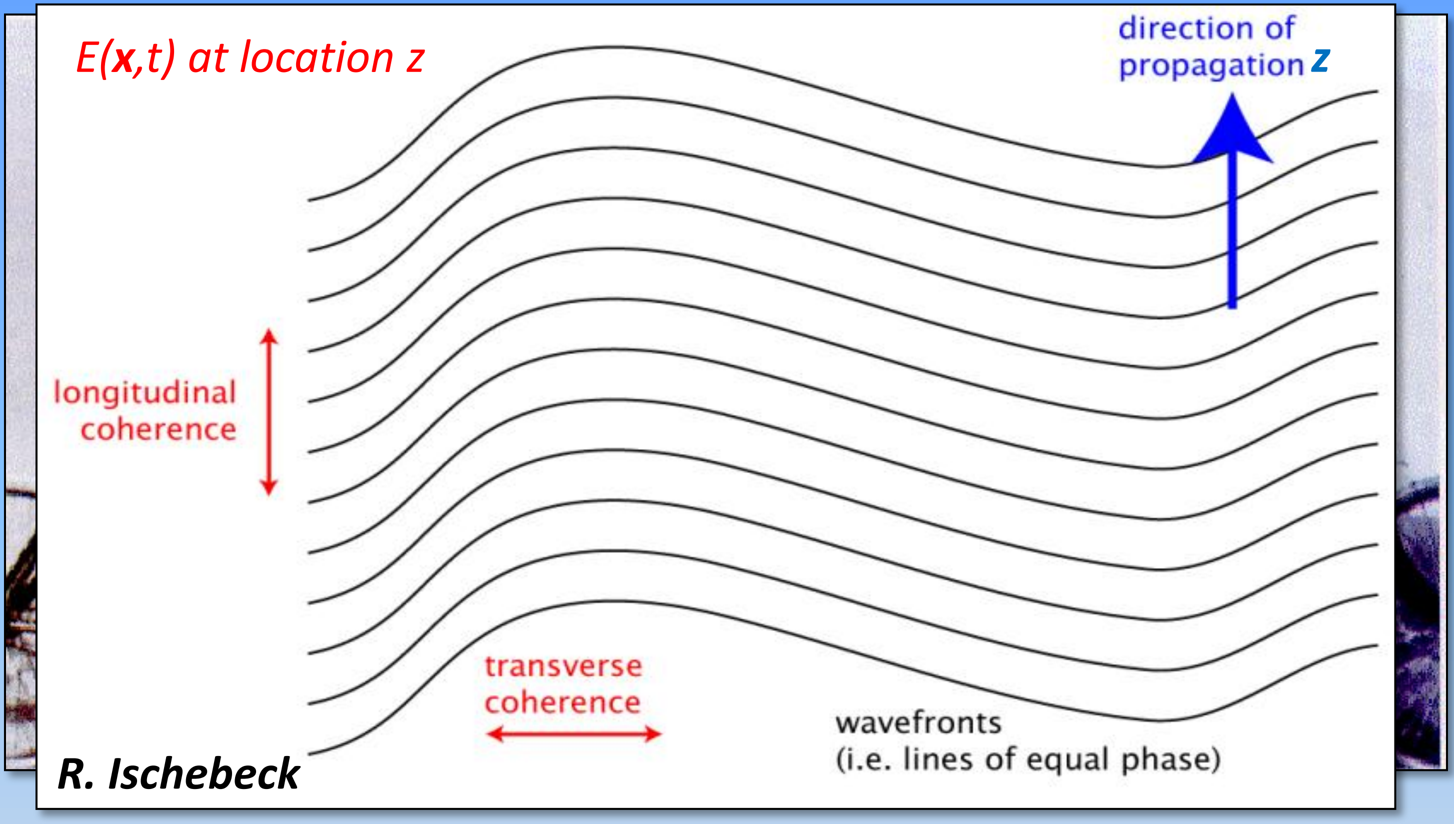
$$\begin{aligned} E(x;z) &= \frac{E_0}{\sqrt{1 + i\sigma_{r'} z/\sigma_r}} \exp\left[-\frac{x^2}{4\sigma_r^2 (1 + i\sigma_{r'} z/\sigma_r)}\right] \\ &= \frac{E_0}{(1 + z^2/Z_R^2)^{1/4}} \exp\left[-\frac{x^2(1 - iz/Z_R)}{4\sigma_r^2(1 + z^2/Z_R^2)} - \frac{i}{2}\tan^{-1}\left(\frac{z}{Z_R}\right)\right] \end{aligned}$$

$$\sigma_r(z) = \sqrt{\frac{\lambda}{4\pi}\left(Z_R + \frac{z^2}{Z_R^2}\right)}$$

- Analogous with electron beam

$$\varepsilon_x \leftrightarrow \frac{\lambda}{4\pi} \qquad \beta_x^* \leftrightarrow Z_R.$$

What is coherence?



- *Complex degree of coherence*

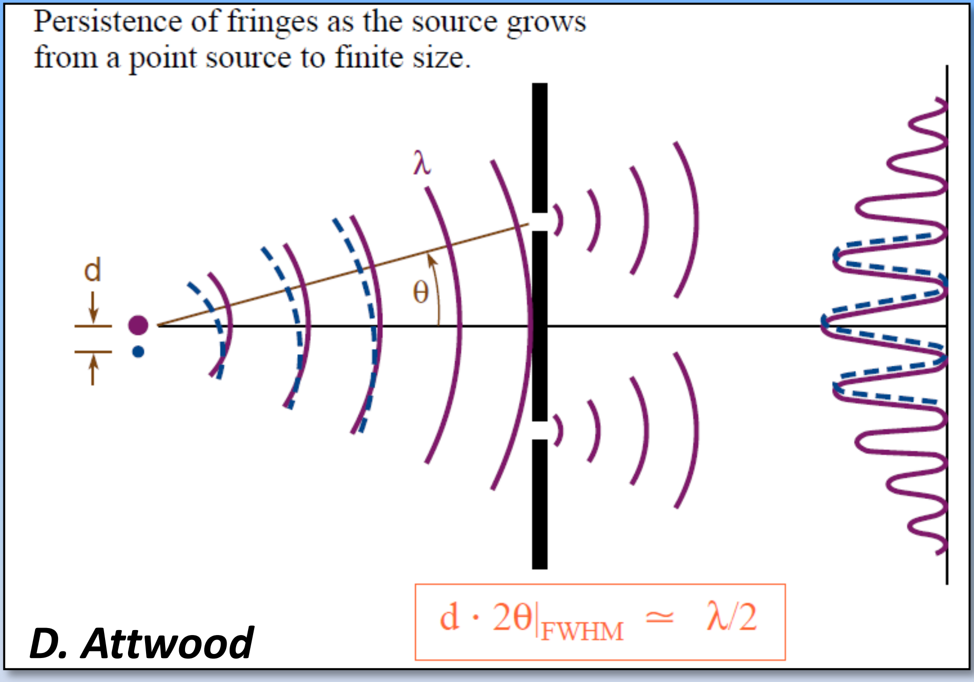
$$\gamma(\boldsymbol{x}_1, \boldsymbol{x}_2, \tau) = \frac{\langle E(\boldsymbol{x}_1, t) E^*(\boldsymbol{x}_2, t+\tau)\rangle}{\sqrt{\langle |E(\boldsymbol{x}_1, t)|^2\rangle \langle |E(\boldsymbol{x}_2, t+\tau)|^2\rangle}}$$

$\gamma(\boldsymbol{x}_1, \boldsymbol{x}_2, 0)$ describes the transverse coherence, $\gamma(0, 0, \tau)$ characterizes the temporal coherence.

Transverse (Spatial) Coherence

- *Transverse coherence can be measured via the interference pattern in Young's double slit experiment.*

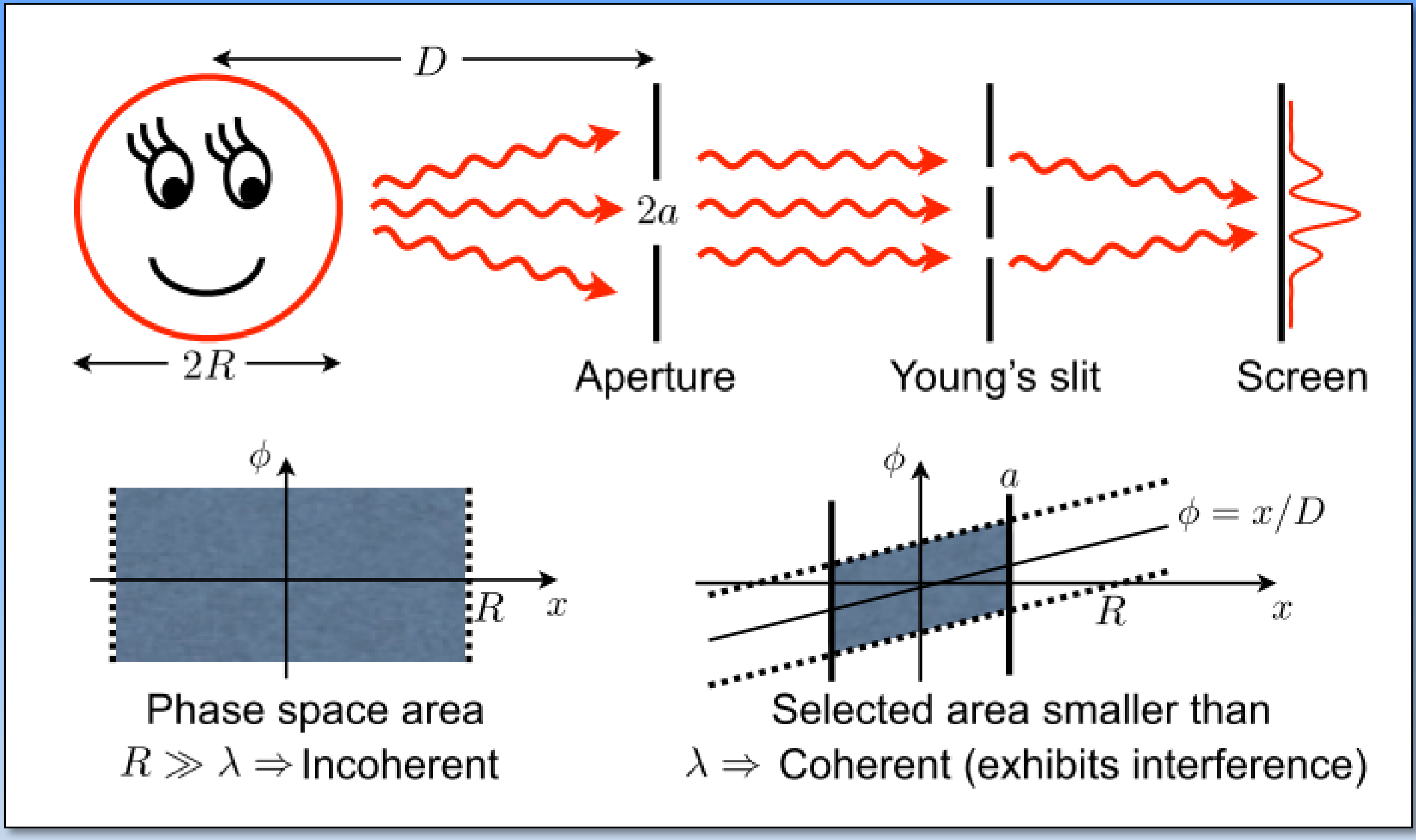
- *Near the center of screen, fringe visibility is described by $\gamma(\mathbf{x}_1,\mathbf{x}_2,0)$.*



- *Degree of transverse coherence (coherence fraction):*

$$\zeta = \frac{\int\int |\gamma(\boldsymbol{x}_1,\boldsymbol{x}_2,0)|^2 I(\boldsymbol{x}_1) I(\boldsymbol{x}_2) d\boldsymbol{x}_1 d\boldsymbol{x}_2}{\int I(\boldsymbol{x}_1) d\boldsymbol{x}_1 \int I(\boldsymbol{x}_2) d\boldsymbol{x}_2}$$

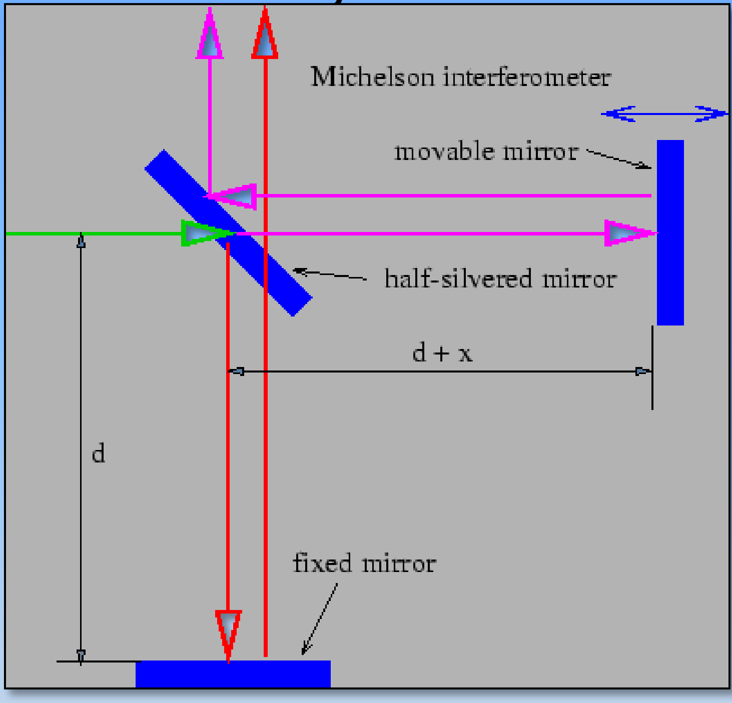
Phase space criteria for transverse coherence



- Initial phase space area $4\pi R >> \lambda$
- Final phase space area $\boxed{4Ra/D \lesssim \lambda/2}$
- Coherent flux is reduced by M_T
- This criteria from physical optics argument

Temporal (Longitudinal) Coherence

- *Coherence time is determined by measuring the path length difference over which fringes can be observed in a Michelson interferometer.*



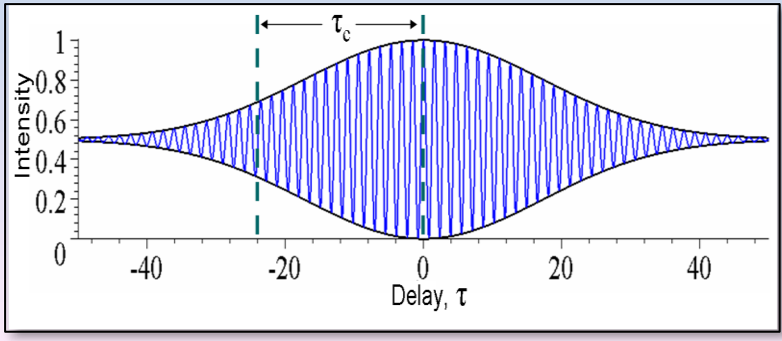
$$\tau_c = \int_{-\infty}^{\infty} d\tau |\gamma(\tau)|^2$$

- *Temporal coherence function and the radiation spectrum forms a Fourier pair*

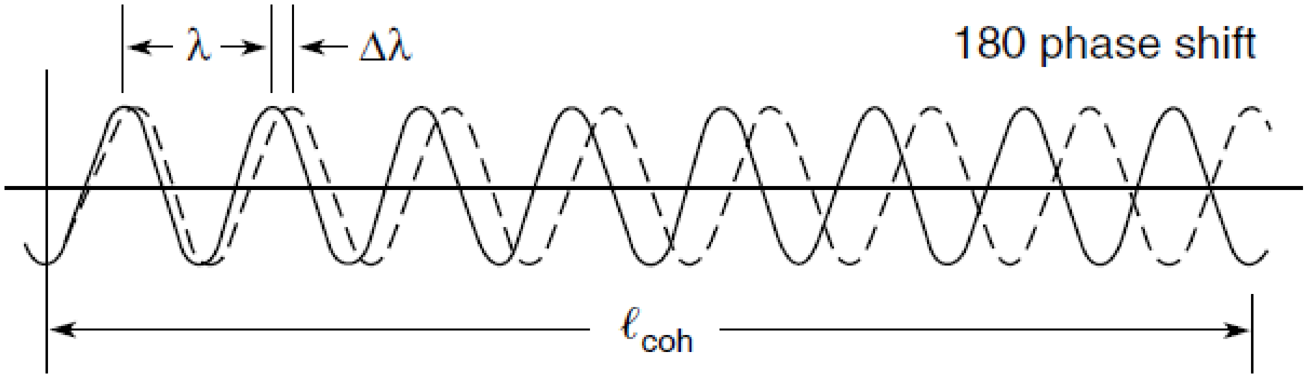
$$\gamma(\tau) = \frac{\int_{-\infty}^{\infty} d\omega |E(\omega)|^2 e^{-i\omega\tau}}{\int_{-\infty}^{\infty} d\omega |E(\omega)|^2}$$

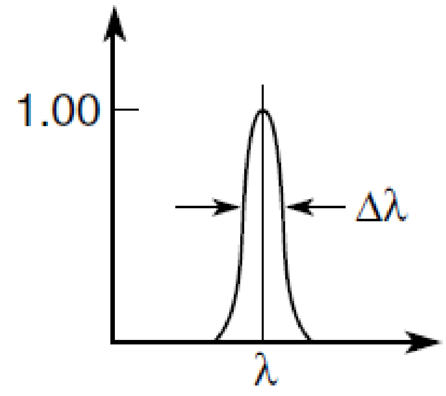
- *For a Gaussian radiation spectrum,*

$$\tau_c = \frac{\sqrt{\pi}}{\sigma_\omega}$$



Temporal Coherence





Define a coherence length ℓ_{coh} as the distance of propagation over which radiation of spectral width $\Delta\lambda$ becomes 180° out of phase. For a wavelength λ propagating through N cycles

$$\ell_{coh} = N\lambda$$

and for a wavelength $\lambda + \Delta\lambda$, a half cycle less $(N - \frac{1}{2})$

$$\ell_{coh} = (N - \tfrac{1}{2})\,(\lambda + \Delta\lambda)$$

Equating the two

$$N = \lambda/2\Delta\lambda$$

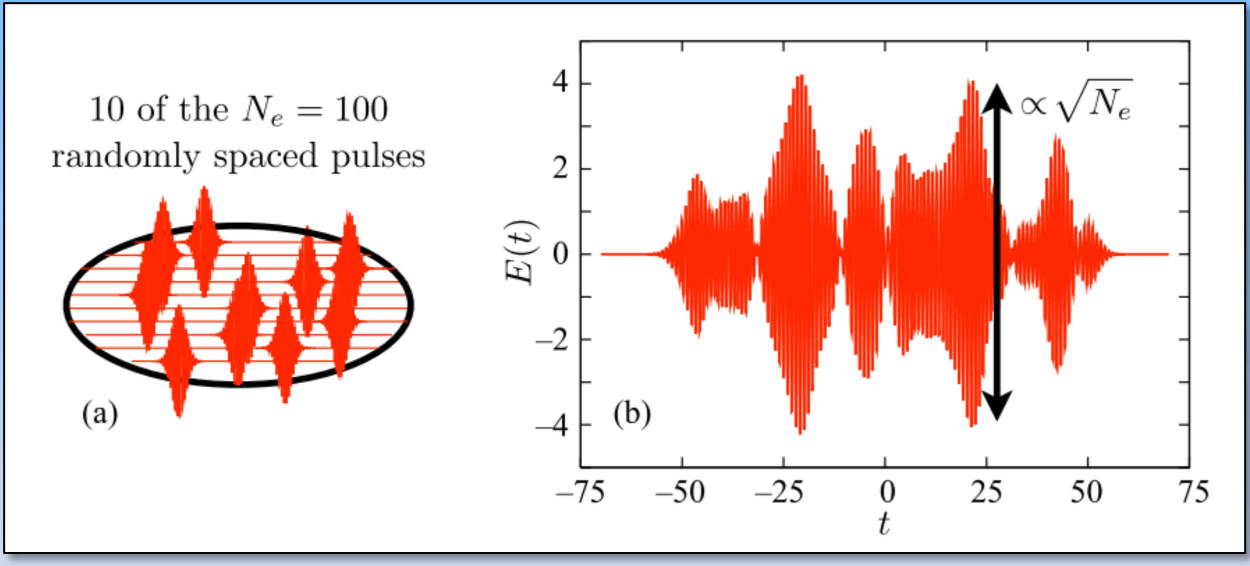
so that

$$\boxed{\ell_{coh} = \frac{\lambda^2}{2\,\Delta\lambda}}$$

Chaotic light

- Radiation from many random emitters (Sun, SR, SASE FEL)

$$E(t) = \sum_{j=1}^{N_e} E_0(t - t_j) = e_0 \sum_{j=1}^{N_e} \exp\left[-\frac{(t-t_j)^2}{4\sigma_\tau^2} - i\omega_1(t - t_j)\right]$$



- Correlation function and coherence time

$$\mathcal{C}(\tau) \equiv \frac{\left\langle \int dt\, E(t)E^*(t+\tau)\right\rangle}{\left\langle \int dt\, |E(t)|^2 \right\rangle}$$

$$t_{\mathrm{coh}} \equiv \int dt\ |\mathcal{C}(\tau)|^2$$

Temporal mode and fluctuation

- Number of regular temporal regions is # of coherent modes

$$M_L \approx \frac{T}{t_{\rm coh}} = \frac{T}{2\sqrt{2\pi}\sigma_\tau} \approx \frac{T}{5\sigma_\tau}.$$

- Intensity fluctuation $\frac{\Delta W}{W} = \frac{1}{\sqrt{M_L}}$

- Same numbers of mode in frequency domain

$$E_\omega = \frac{e_0\sigma_\tau}{\sqrt{\pi}} \sum_{j=1}^{N_e} \exp\left[-\frac{(\omega-\omega_1)^2}{4\sigma_\omega^2} + i\omega t_j\right]$$

$$c\sigma_\tau \cdot \frac{\sigma_\omega}{\omega_1} = \frac{\lambda_1}{4\pi},$$

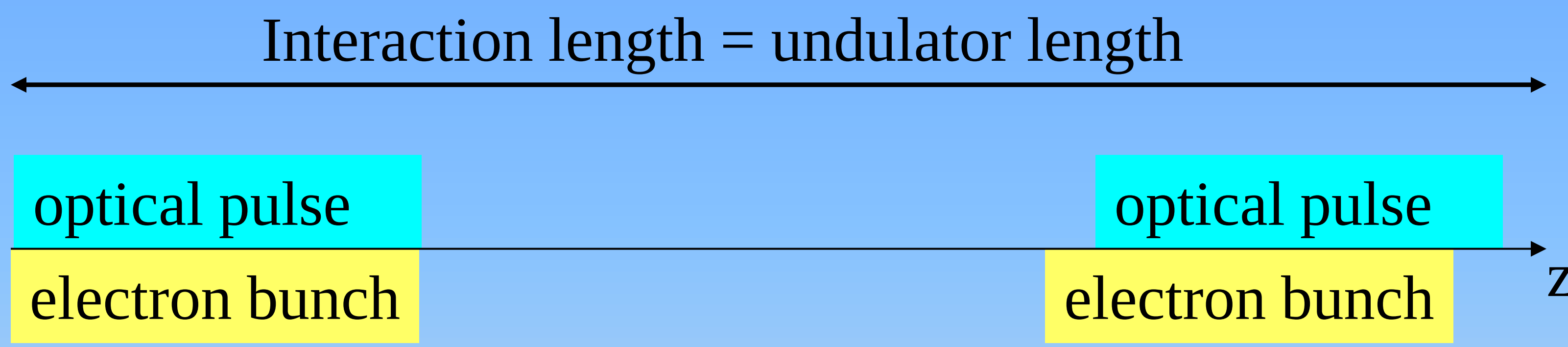
Fourier limit, minimum longitudinal phase space

- Longitudinal phase space is M_L larger than Fourier limit

- Total # of modes $M = M_L M_T^2.$

Projected and slice beam parameters

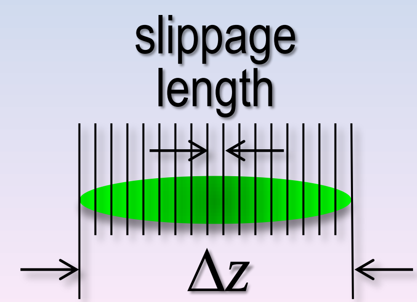
- Due to resonant condition, light overtakes e-beam by one radiation wavelength λ_1 per undulator period



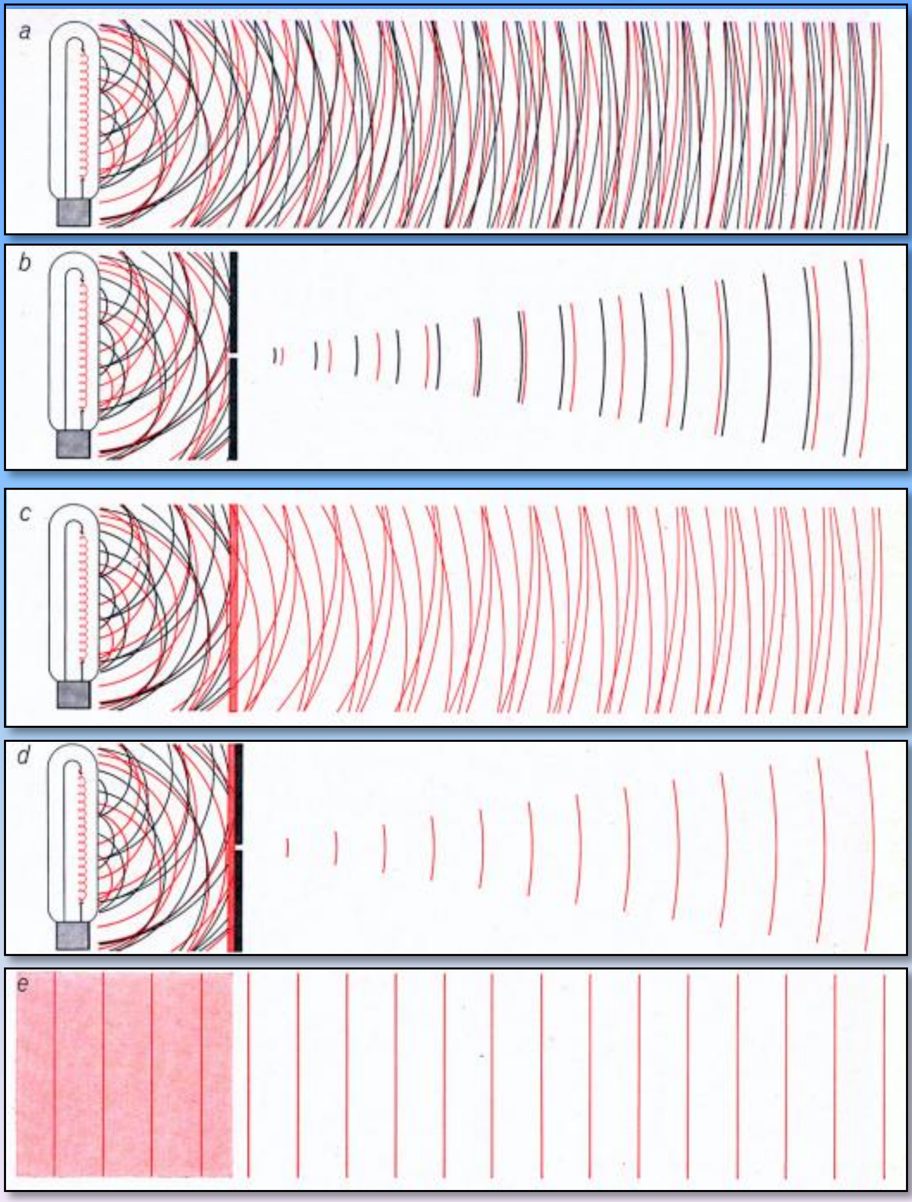
Slippage length = $\lambda_1 \times$ undulator period
(e. g., 100 m LCLS undulator has slippage length 1.5 fs, much less than 100-fs e-bunch length)

- Each part of optical pulse is amplified by those electrons within a slippage length (an FEL slice)

- Only slices with good beam qualities (emittance, current, energy spread) can lase



Light Bulb vs. Laser



Radiation emitted from light bulb is chaotic.

Pinhole can be used to obtain spatial coherence.

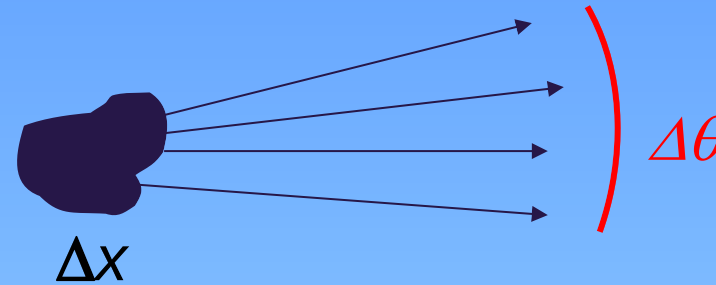
Monochromator can be used to obtain temporal coherence.

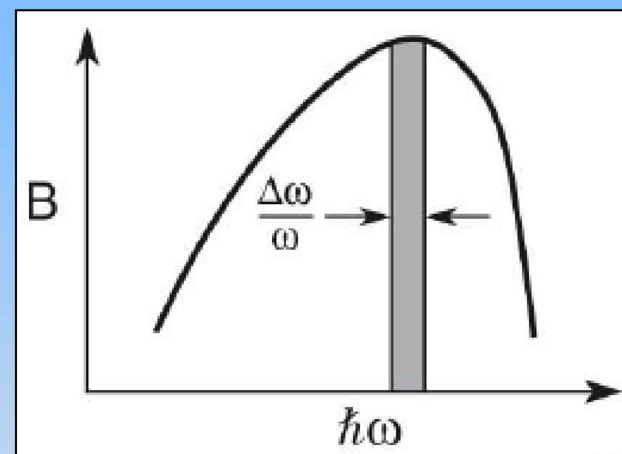
Pinhole and Monochromator can be combined for coherence.

Laser light is spatially and temporally coherent.

A. Schawlow (**Nobel prize on laser spectroscopy**), Scientific Americans, 1968

Brightness





$$B = \frac{\text{Photons in unit spectral range in unit time}}{(\text{source size} \times \text{divergence})^2} \rightarrow \begin{cases} \textbf{Peak} \\ \textbf{Average} \end{cases}$$

Units: photons/s/mm²/mrad²/0.1%BW

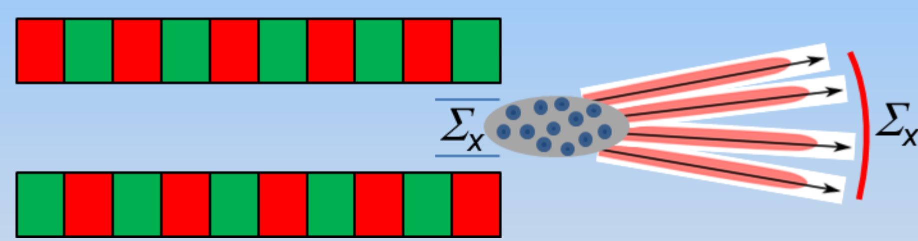
Brightness via Wigner Function

- *Spectral brightness defined via Wigner function, which is Fourier transformation of the transverse correlation function (K.J. Kim, 1986).*

$$B(\boldsymbol{x},\boldsymbol{\phi};z)=\frac{d\omega}{\hbar\omega}\frac{\omega^2\varepsilon_0}{\pi c T}\int d\boldsymbol{\xi}e^{ik\boldsymbol{\xi}\cdot\boldsymbol{\phi}}\left\langle E\left(\boldsymbol{x}+\tfrac{1}{2}\boldsymbol{\xi};z\right)E^*\left(\boldsymbol{x}-\tfrac{1}{2}\boldsymbol{\xi};z\right)\right\rangle$$

- *Brightness is conserved in a perfect optical system: cannot increase brightness once the source is born.*

- *Brightness convolution theorem*



$$B(\boldsymbol{x},\boldsymbol{\phi},z)=N_e\int d\boldsymbol{x}_j d\boldsymbol{x}'_j B_j(\boldsymbol{x}-\boldsymbol{x}_j,\boldsymbol{\phi}-\boldsymbol{x}'_j,z)f(\boldsymbol{x}_j,\boldsymbol{x}'_j,z)$$

single electron rad. brightness ***electron distribution function***

Radiation from many electrons

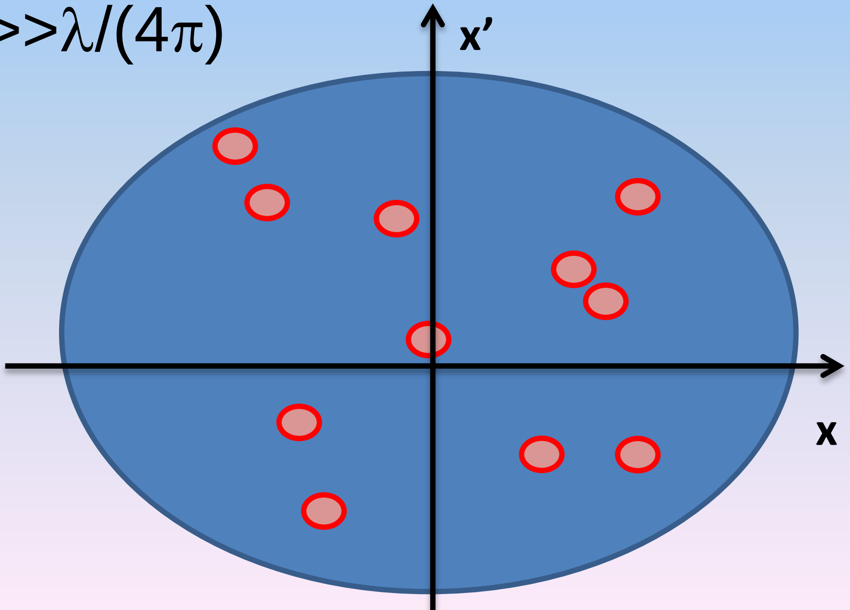
- Such a beam can be described by the convolution of the coherent Gaussian beam with the electron distribution in phase space

- Same formula as previous slide except $\sigma_r \rightarrow \Sigma_r$, $\sigma_{r'} \rightarrow \Sigma_{r'}$

$$\Sigma_x = \sqrt{\sigma_x^2 + \sigma_r^2} \qquad \Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{r'}^2}.$$

- When electron beam emittance $>>\lambda/(4\pi)$

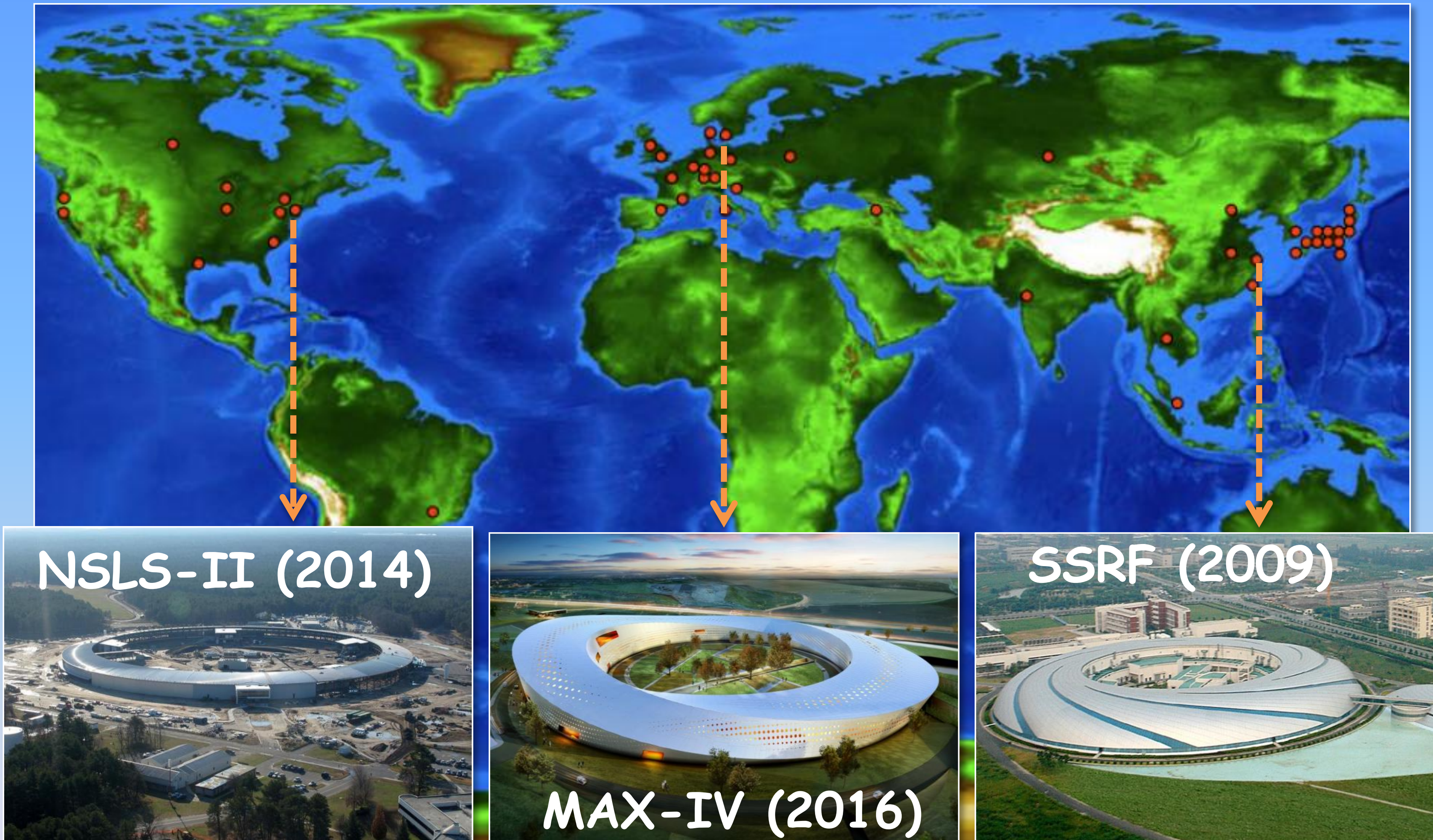
$$\Sigma_x \Sigma_{x'} \gg \frac{\lambda}{4\pi}$$



- # of transverse modes

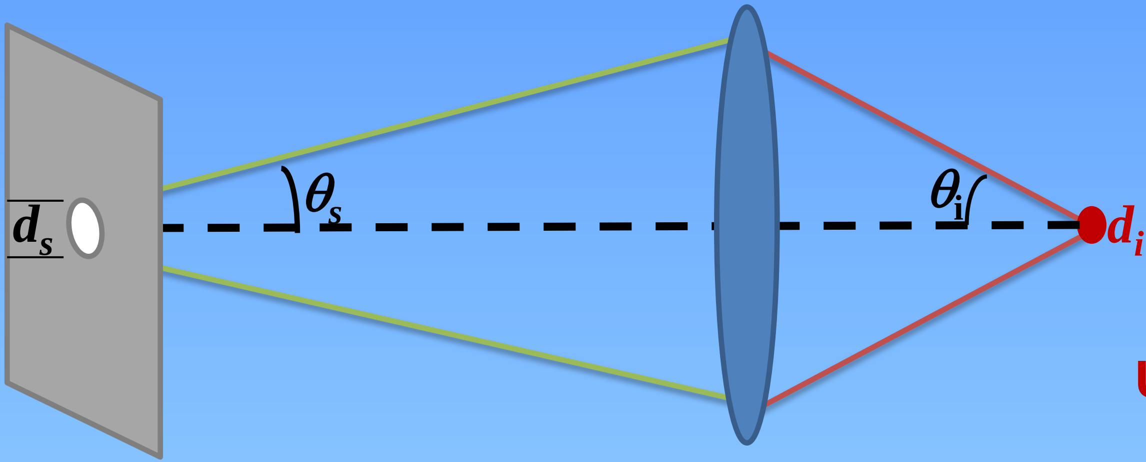
$$M_T = \frac{\Sigma_x \Sigma_{x'}}{\lambda/4\pi} = \frac{\varepsilon_x}{\varepsilon_r}$$

Synchrotron Radiation Facilities



- *State-of-art storage rings have **pulse duration ~10 ps, emittance ~1 nm.***
- *Diffraction-limited storage rings and energy recovery linacs with **emittance ~10 pm** are under active R&D.*

Diffraction Limit

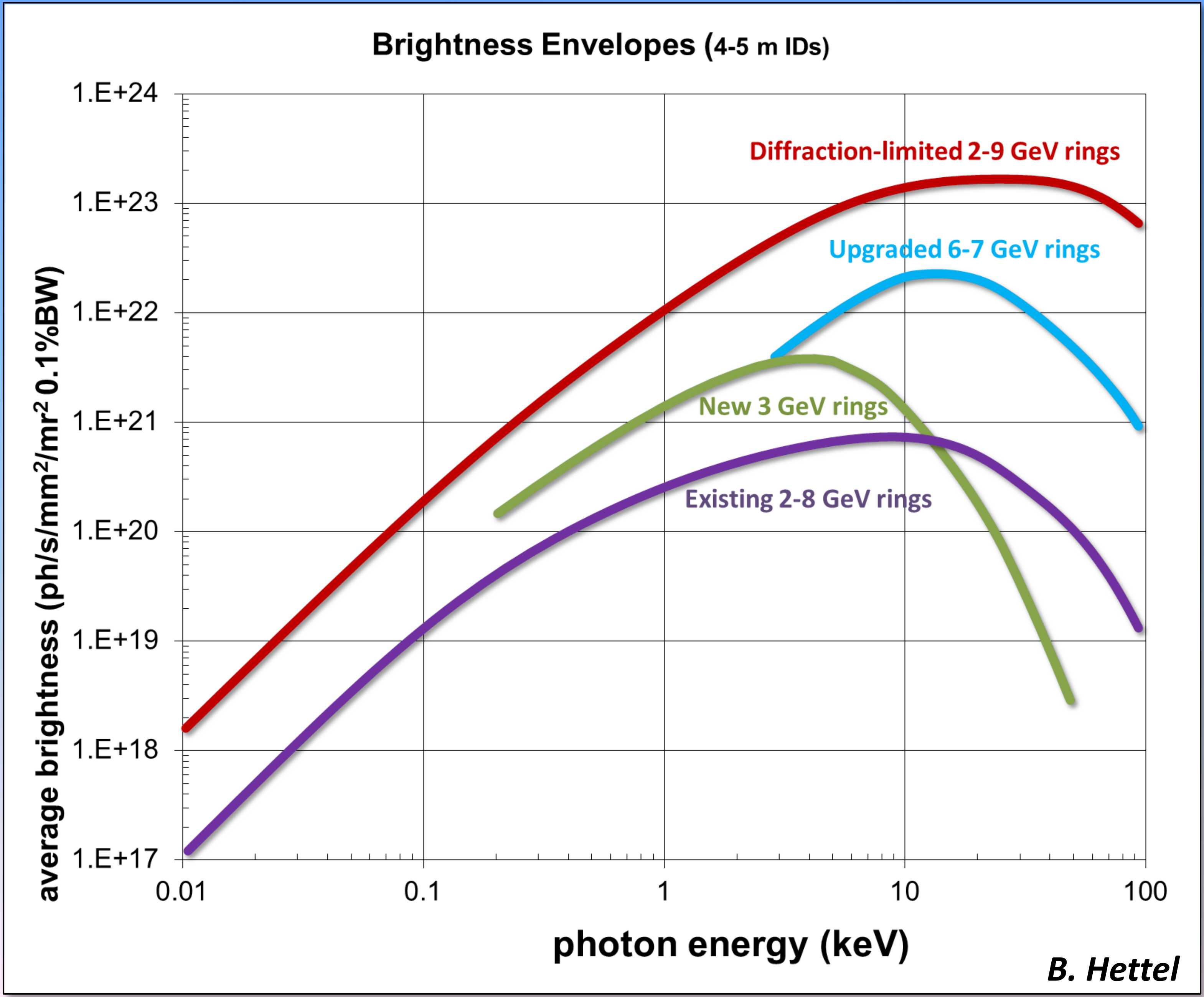


$$\varepsilon_x \sim \varepsilon_y \sim \frac{\lambda_r}{4\pi}$$

Ultimate spatial resolution

- *Perfect optical system has* $\mathbf{d_s}\boldsymbol{\theta_s} = \mathbf{d_i}\boldsymbol{\theta_i}$
  $\boldsymbol{\theta_i}$ *is the numerical aperture of focusing system*
- *Reducing pinhole size until* $\mathbf{d_s}\boldsymbol{\theta_s} \sim \lambda/2$
  since $\mathbf{d_i} \sim \lambda/(2\theta_i)$ *reaches diffraction limit.*
- *A even smaller pinhole does not reduce the image size but only hurts the photon flux*
- ***Diffraction limited source does not require a pinhole and provide the most coherent flux***

Storage Ring Spectral Brightness



Radiation intensity

- What if emitters are not random in time

$$\left\langle |E(\omega)|^2 \right\rangle = |E_\omega^0|^2 \left\langle \left| \sum_{j=1}^{N_e} e^{i\omega t_j} \right|^2 \right\rangle$$

$$\left\langle \left| \sum_{j=1}^{N_e} e^{i\omega t_j} \right|^2 \right\rangle = N_e + \left\langle \sum_{j\neq k}^{N_e} e^{i\omega (t_j - t_k)} \right\rangle$$

$$\left\langle \left| \sum_{j\neq k}^{N_e} e^{i\omega (t_j - t_k)} \right|^2 \right\rangle = N_e(N_e - 1) \left| \int dt\ f(t) e^{i\omega t} \right|^2$$

- For an electron bunch with rms bunch length σ_e

$$f(t) = \frac{1}{\sqrt{2\pi}\,\sigma_e} \exp\left(-\frac{t^2}{2\sigma_e^2} \right)$$

$$\left\langle |E(\omega)|^2 \right\rangle = N_e |E_\omega^0|^2 \left[1 + (N_e - 1) e^{-\omega^2 \sigma_e^2} \right]$$

- When $(N_e - 1) e^{-\omega^2 \sigma_e^2} \ll 1$

  intensity from many electrons add **incoherently (~N_e)**

Bunching and coherent radiation

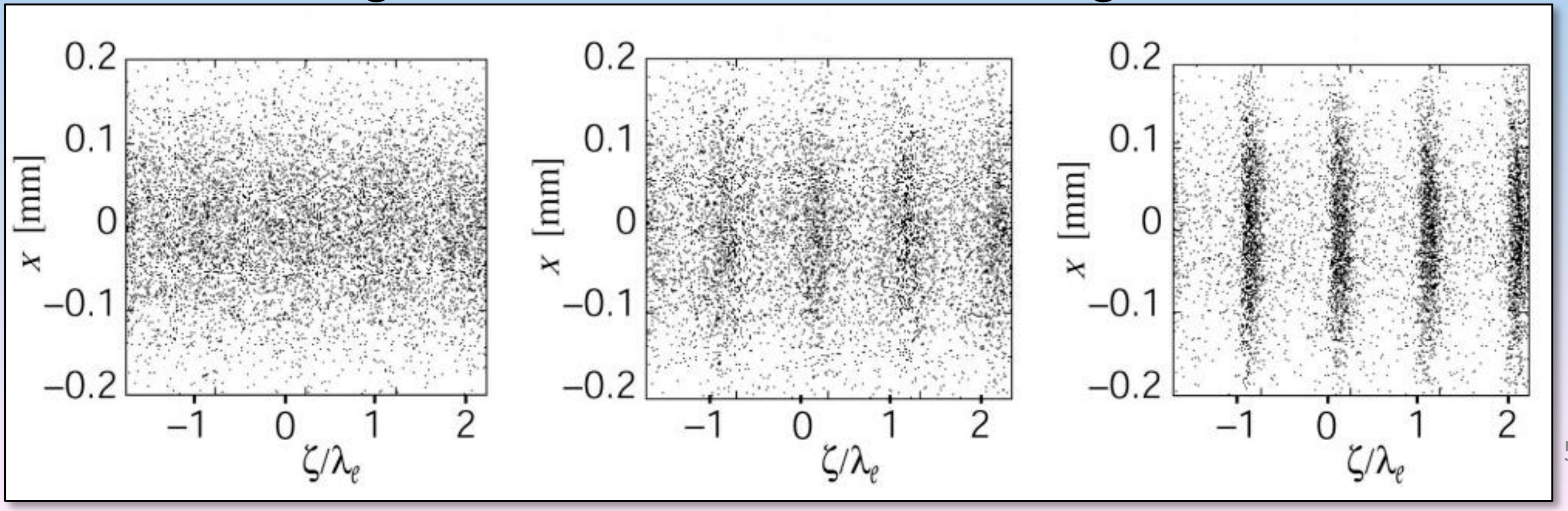
- If the bunch length is shorter than the radiation wavelength

$$\boxed{(N_e - 1)e^{-\omega^2\sigma_e^2} \geq 1}$$

$$\left\langle \left|E(\omega)\right|^2 \right\rangle = N_e \left|E_\omega^0\right|^2 \left(1 + (N_e - 1)\left|f(\omega)\right|^2\right)$$

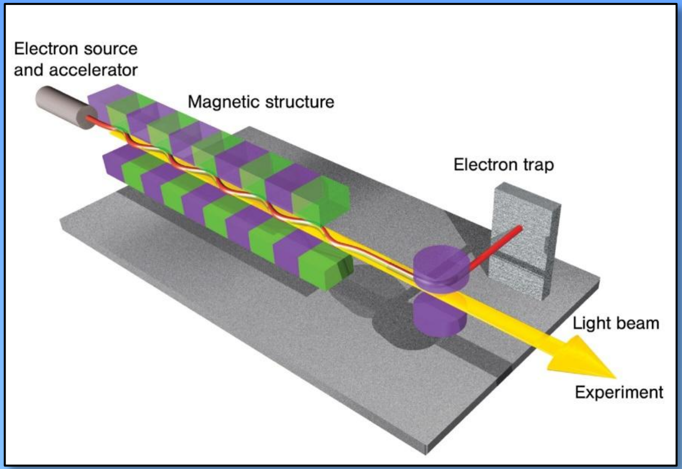
Form factor or bunching factor

- Radiation intensity from many electrons add **coherently (~N_e^2)**
- Another way to produce bunching from a relatively long bunch is through so-called microbunching

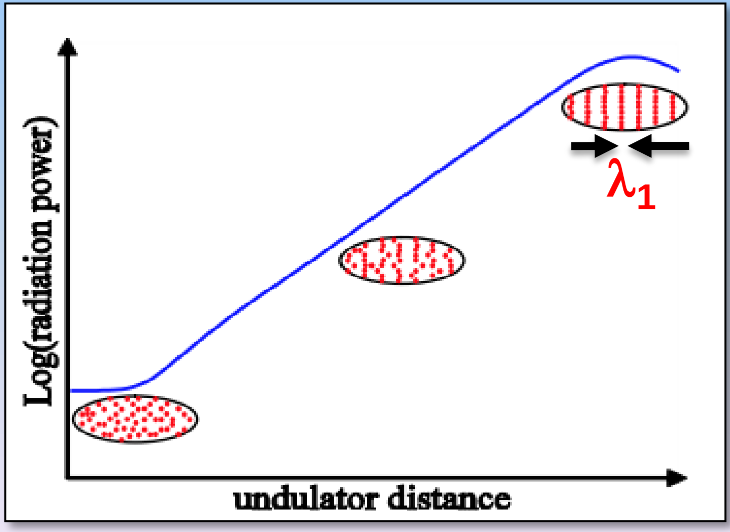


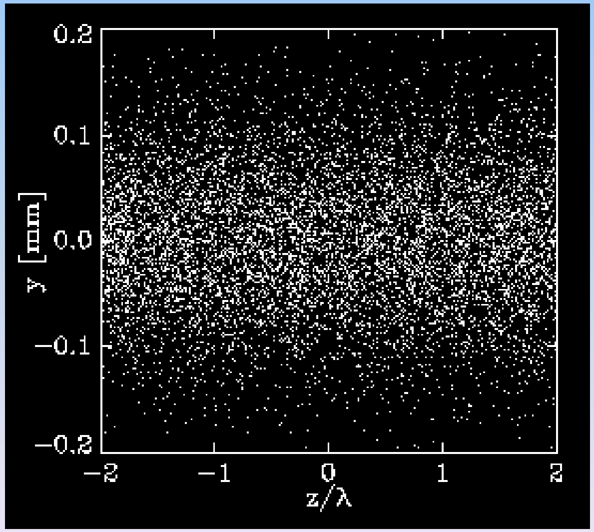
Free Electron Laser (FEL)

- ***Resonant interaction** of electrons with EM radiation in an undulator^*
- ***Coherent radiation intensity $\propto N^2$ due to beam microbunching***
  (N: # of e^- involved ~10^6 to 10^9)



- *At x-ray wavelengths, use **Self-Amplified Spontaneous Emission*** **(a wonderful instability!)** to reach high peak power*





S. Reiche

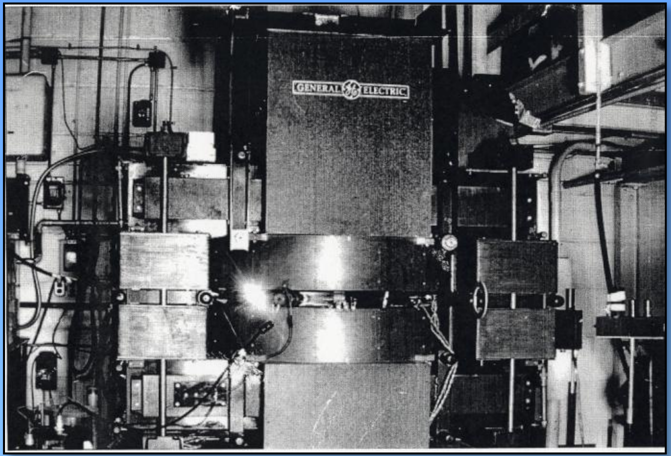
^ J. Madey, J. Appl. Phys., 1971

** Kondradenko, Saldin, Part. Accel., 1980*

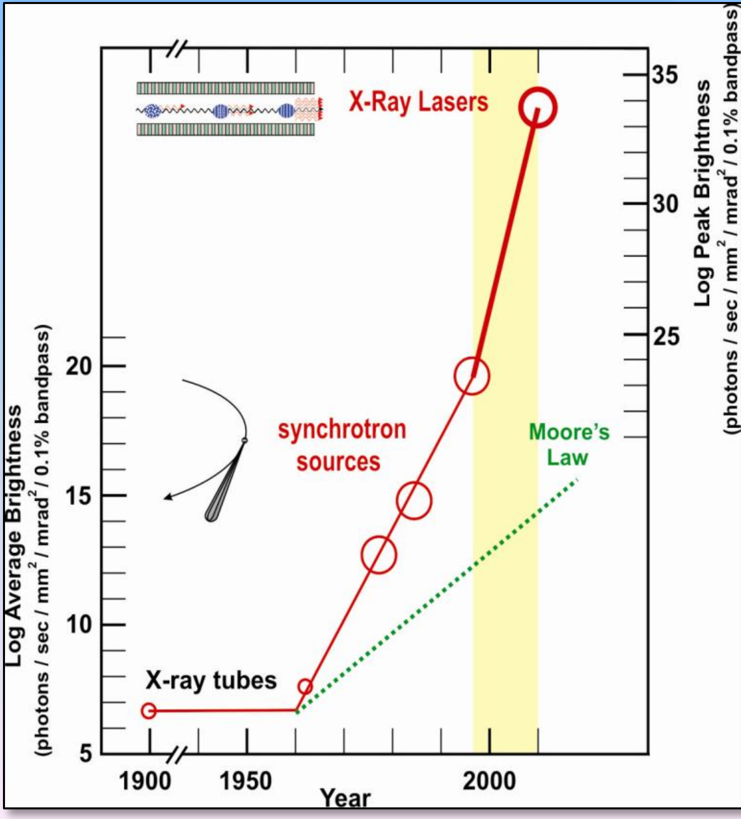
** Bonifacio, Pellegrini, Narducci, Opt. Com., 1984*

Evolution of X-ray Light Sources

- *GE synchrotron (1946) opened a new era of accelerator-based light sources.*

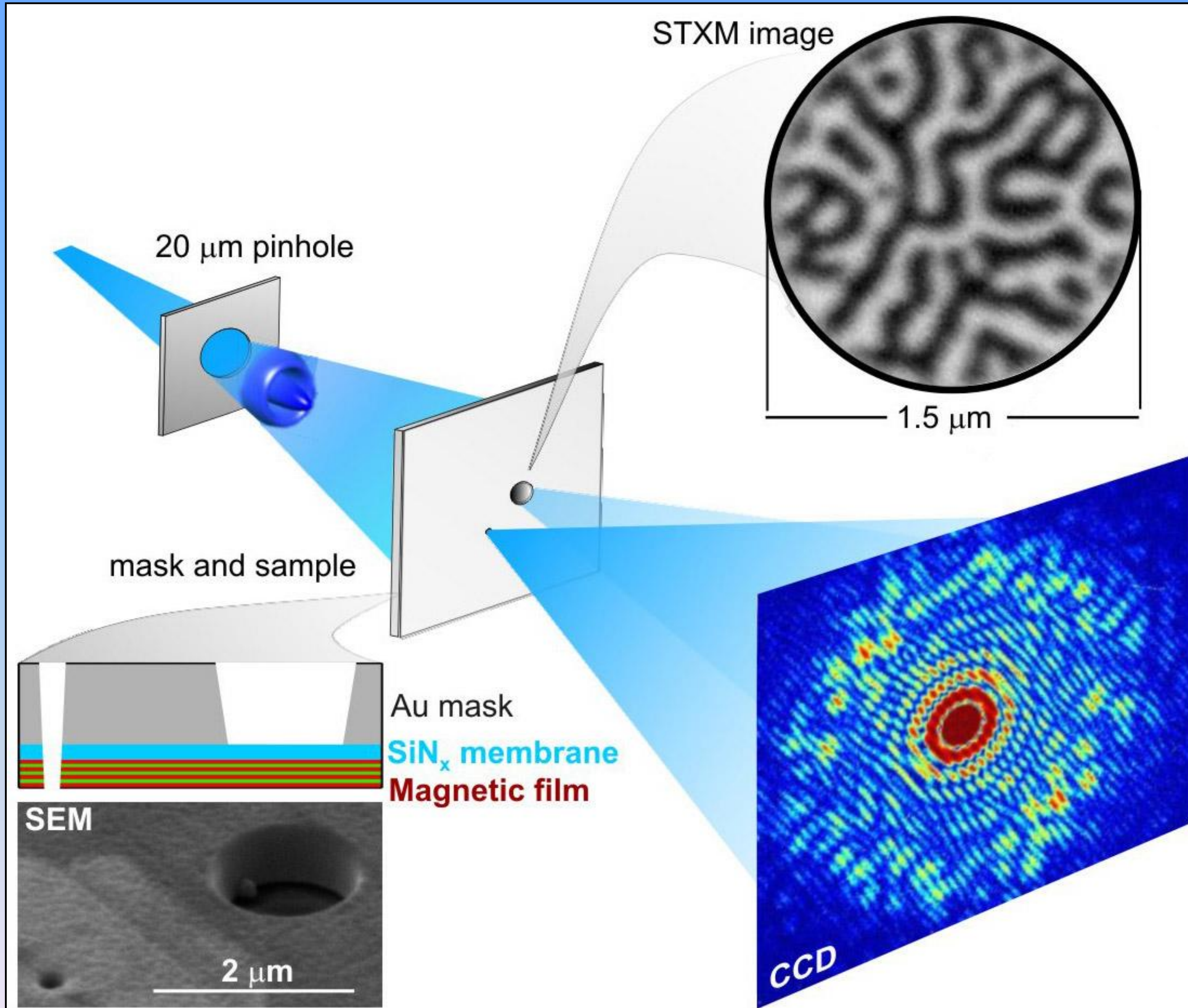


- *These light sources have evolved rapidly over four generations.*
  - *The first three-generations are based on synchrotron radiation.*
  - *The forth-generation light source is a game-changer based on FELs.*
- *The dramatic improvement of brightness and coherence over 60 years easily outran Moore's law.*



X-Ray Holography: Coherence Wanted

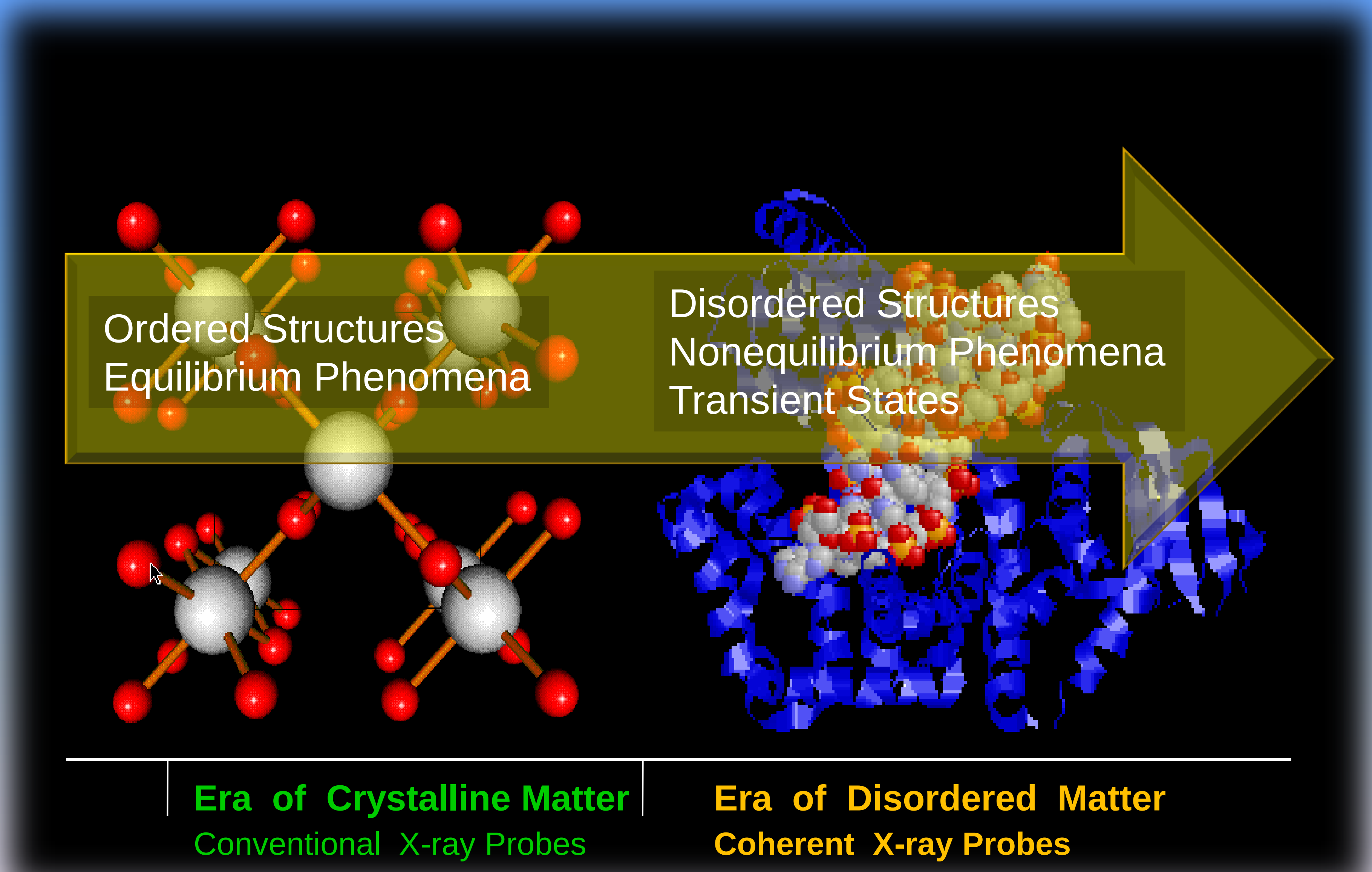
Lensless imaging of magnetic nanostructures by x-ray holography



S. Eisebitt, J. Lüning, W.F. Schlotter, M. Lörgen, O. Hellwig,
W. Eberhardt & J. Stöhr / *Nature*, 16 Dec 2004

LenslessImagingF1.ai

Future Role of FELs and Advanced Sources



1900 *2000* *future*

H. Dosch (DESY)

Summary

- ***Despite spectacular successes in synchrotron radiation and FELs, the quest for brightness and coherence continues, with no sign of slowing down.***

- ***Future light source development includes diffraction-limited light sources, high-peak and average power FELs, compact coherent sources and many more possibilities.***

- ***I hope you enjoy this summer school and this exciting field of research.***