Basic concepts in electron and photon beams

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July 22, 2013
Lecture Outline

Electron beams

Photon beams

References:

2. Helmut Wiedemann, Particle Accelerator Physics (Springer-Verlag, 2003).
8. Images from various sources and web sites.
Motivations:
Why does anyone care about accelerators?

Exciting products… exciting opportunities
Electron beams

- Primer on special relativity and E&M
- Accelerating electrons
- Transporting electrons
- Beam emittance and optics
- Beam distribution function
Lorentz Transformation

\[
\begin{align*}
\begin{pmatrix}
x \\
y \\
z \\
c \tau
\end{pmatrix}
&= 
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \gamma & \beta \gamma \\
0 & 0 & \beta \gamma & \gamma
\end{pmatrix}
\begin{pmatrix}
x^* \\
y^* \\
z^* \\
c \tau^*
\end{pmatrix}
\end{align*}
\]

\[
\beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \beta = \sqrt{1 - \frac{1}{\gamma^2}}
\]
Length Contraction and Time Dilation

- Length contraction: an object of length $\Delta z'$ aligned in the moving frame with the $z'$ axis will have the length $\Delta z$ in the lab frame
  \[ \Delta z = \frac{\Delta z'}{\gamma} \]

- Time dilation: Two events occurring in the moving frame at the same point and separated by the time interval $\Delta t'$ will be measured by the lab observers as separated by $\Delta t$
  \[ \Delta t = \gamma \Delta t' \]
Energy, Mass, Momentum

Energy

\[ E = T + mc^2 \]

Electrons rest mass energy 511 keV (938 MeV for protons),
\( 1 \text{eV} = 1.6 \times 10^{-19} \text{Joule} \)

Momentum

\[ p = \gamma \beta mc \]

Energy and momentum

\[ E^2 = p^2 c^2 + m^2 c^4, \quad E = \gamma mc^2. \]
Maxwell’s Equations

\[ \nabla \cdot \mathbf{D} = \rho \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \]

\[ D = \varepsilon_0 E \]
\[ B = \mu_0 H \]

\[ c = (\varepsilon_0 \mu_0)^{-1/2} \]

\[ Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377 \text{ Ohm} \]

- Wave equation

\[
\left( \frac{\partial^2}{\partial t^2} - c^2 \nabla^2 \right) \mathbf{E} = -\frac{1}{\varepsilon_0} \left( \frac{\partial \mathbf{j}}{\partial t} + c^2 \nabla \rho \right)
\]

- Lorentz transformation of fields

\[
E_z = E'_z, \quad E_\perp = \gamma \left( E'_\perp - \mathbf{v} \times \mathbf{B}' \right),
\]
\[
B_z = B'_z, \quad B_\perp = \gamma \left( B'_\perp + \frac{1}{c^2} \mathbf{v} \times \mathbf{E}' \right)
\]
Lorentz Force

- Lorentz force

\[ \mathbf{F} = e\mathbf{E} + e\mathbf{v} \times \mathbf{B} \]

- Momentum and energy change

\[ \Delta \mathbf{p} = \int \mathbf{F} \, dt \]
\[ \Delta E = \int \mathbf{F} \, ds \quad \text{with} \quad ds = \mathbf{v} \, dt \]

- Energy exchange through \( E \) field only

\[ \Delta E = \int \mathbf{F} \, ds = e \int \mathbf{E} \cdot ds + e \int (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} \, dt \]

\[ = 0 \]

No work done by magnetic field!
A relativistic electron

In electron’s frame, Coulomb field is

\[ E' = \frac{1}{4\pi\varepsilon_0} \frac{e\gamma x}{r'^3} \]

In lab frame, space charge field is

\[ E_x = \frac{1}{4\pi\varepsilon_0} \frac{e\gamma x}{[x^2 + y^2 + \gamma^2(z - vt)^2]^{3/2}} \]
\[ E_y = \frac{1}{4\pi\varepsilon_0} \frac{e\gamma y}{[x^2 + y^2 + \gamma^2(z - vt)^2]^{3/2}} \]
\[ E_z = \frac{1}{4\pi\varepsilon_0} \frac{e\gamma(z - vt)}{[x^2 + y^2 + \gamma^2(z - vt)^2]^{3/2}} \]
Guiding beam: dipole

- Lorentz force
  \[ F = eE + e\mathbf{v} \times \mathbf{B} \]

- Centrifugal force
  \[ F_{cf} = \frac{\gamma mc^2 \beta^2}{\rho} \]

- Bending radius is obtained by balance the forces
  \[ \frac{1}{\rho} = \frac{eB}{\gamma \beta mc^2} \]
  \[ \frac{1}{\rho} \text{[m}^{-1}] = 0.2998 \frac{B[\text{T}]}{\beta E[\text{GeV}]} \]
Cyclotron

If beam moves circularly, re-traverses the same accelerating section again and again, we can accelerate the beam repetitively.
Lawrence started to construct a cyclotron, as the machine later was named, in early 1930. A graduate student, M. Stanley Livingston, did much of the work of translating the idea into working hardware. In January 1931 Lawrence and Livingston met their first success. A device about 4.5 inches in diameter used a potential of 1,800 volts to accelerate hydrogen ions up to energies of 80,000 electron volts. Lawrence immediately started planning for a bigger machine. In summer 1931 an eleven-inch cyclotron achieved a million volts.

"Dr Livingston has asked me to advise you that he has obtained 1,100,000 volt protons. He also suggested that I add 'Whoopee'!"

—Telegram to Lawrence, 3 August 1931

http://www.aip.org/history/lawrence/first.htm
From Cyclotron to Synchrotron

- Cyclotron does not work for relativistic beams.

- **Cyclotron**
  - huge dipole, compact design,
  - $B = \text{constant}$
  - low energy, single pass.

- **Synchrotron**
  - varying $B$, small magnets, high energy
GE synchrotron observed first synchrotron radiation (1946) and opened a new era of accelerator-based light sources.

The first purpose-built synchrotron to operate was built with a glass vacuum chamber.
Electron linac

The rf energy is used to launch a traveling wave or standing wave in an array of cavities.

\[ V = K \sqrt{P_0 \ell r_s} , \text{ where } K < 1 \]

The linac structure is designed such that the phase of the EM wave is synchronized with the beam, i.e. \( v_p \sim c \). In a smooth waveguide, the phase velocity \( v_p > c \). Those disks are used to slow down the waveguide phase velocity in order to achieve synchronism with the electron beams.
Electron Linac (disk loaded structure)

[toward the right] +

Electric Field

[toward the left] -

Positive particles

behind the bunch
on time
ahead

amount of energy boost

Position

Negative particles

ahead
on time
behind

1/20,000,000,000 second later
(notice how far the bunches have moved)

[Ref.] http://www.slac.stanford.edu
Disk loaded structure made at Stanford Univ. (1947)

Historical Image

STANFORD LINEAR ELECTRON ACCELERATOR PROJECT, REPORT NO. SEVEN

We have accelerated electrons.
Stanford Linear Accelerator Center (SLAC)

50 x 50 GeV e-e+
**Linac Coherent Light Source (LCLS) at SLAC**

X-FEL based on last 1-km of existing 3-km linac

Proposed by C. Pellegrini in 1992

1.5-15 Å
(14-4.3 GeV)

Existing 1/3 Linac (1 km)
(with modifications)

New $e^-$ Transfer Line (340 m)

X-ray Transport Line (200 m)

Undulator (130 m)

Near Experiment Hall

Far Experiment Hall

Injector (35°) at 2-km point
Beam description

Beam phase space \((x, x', y, y', \Delta t, \Delta \gamma)\)

\[ x' \equiv \frac{dx}{dz} = \frac{dx}{dt} / \frac{dz}{dt} = \frac{1}{v_z} \frac{dx}{dt} \]

\[ \Delta \gamma_j \equiv \gamma_j - \gamma_0 \]

Consider paraxial beams such that

\[ |x'| = \sqrt{x'^2 + y'^2} \approx \frac{1}{c} \sqrt{v_x^2 + v_y^2} \ll 1 \]
Linear beam transport

- Transport matrix

\[
\begin{bmatrix}
  x \\
  x'
\end{bmatrix}_o = M(z_i, z_0) \begin{bmatrix}
  x \\
  x'
\end{bmatrix}_i
\]

- Free space drift

\[
\begin{bmatrix}
  x \\
  x'
\end{bmatrix}_o = \begin{bmatrix}
  1 & \ell \\
  0 & 1
\end{bmatrix} \begin{bmatrix}
  x \\
  x'
\end{bmatrix}_i \equiv M_\ell \begin{bmatrix}
  x \\
  x'
\end{bmatrix}_i
\]

- Quadrupole (de-)focusing

\[
\begin{bmatrix}
  x \\
  x' \\
  y \\
  y'
\end{bmatrix}_o = M_f \begin{bmatrix}
  x \\
  x' \\
  y \\
  y'
\end{bmatrix}_i = \begin{bmatrix}
  1 & 0 & 0 & 0 & 0 \\
  -1/f & 1 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 1/f & 1
\end{bmatrix} \begin{bmatrix}
  x \\
  x' \\
  y \\
  y'
\end{bmatrix}_i
\]
Beam properties

- Second moments of beam distribution
  - rms size
  - rms divergence
  - correlation
**Beam emittance**

- **Emittance or geometric emittance**

\[ \varepsilon_x \equiv \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle} \]

- Emittance is **conserved** in a linear transport system

- **Normalized emittance** is conserved in a linear system including acceleration

\[ \varepsilon_{x,n} = \beta_z \gamma \varepsilon_x \approx \gamma \varepsilon_x \]

- Normalized emittance is hence an important figure of merit for electron sources

- Preservation of emittances is critical for accelerator designs.
Beam optics function

- Optics functions (Twiss parameters)

\[
\beta_x = \frac{\langle x^2 \rangle}{\varepsilon_x}, \quad \gamma_x = \frac{\langle x'^2 \rangle}{\varepsilon_x}, \quad \alpha_x = -\frac{\langle xx' \rangle}{\varepsilon_x}
\]

- Given beta function along beamline

\[
\beta_x \gamma_x - \alpha_x^2 = 1
\]

\[
\sigma_x(z) = \sqrt{\varepsilon_x \beta_x(z)}
\]
Free space propagation

- **Single particle**

\[
\begin{bmatrix} x \\ x' \end{bmatrix}_o = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix}_i
\]

- **Beam envelope**

\[\langle x_o^2 \rangle = \langle (x_i + z x'_i)^2 \rangle = \langle x_i^2 \rangle + 2z \langle x_i x'_i \rangle + z^2 \langle x'_i^2 \rangle\]

\[\beta_x(z) = \beta_x(0) + z^2 \gamma_x(0)\]

\[\sigma_x(z) = \sqrt{\varepsilon_x \beta_x(z)} = \sqrt{\varepsilon_x \left( \beta_x^* + \frac{z^2}{\beta_x^*} \right)}\]

- **Analogous with Gaussian laser beam**

\[\varepsilon_x \leftrightarrow \frac{\lambda}{4\pi}\]

\[\beta_x^* \leftrightarrow Z_R.\]
FODO lattice

- Multiple elements (e.g., FODO lattice)

\[ M = M_N M_{N-1} \ldots M_2 M_1 \]

\[
\begin{align*}
M_{\text{FODO}} &= \begin{bmatrix}
1 & 0 \\
-1/2f & 1
\end{bmatrix}
\begin{bmatrix}
1 & \ell \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
1/f & 1
\end{bmatrix}
\begin{bmatrix}
1 & \ell \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
-1/2f & 1
\end{bmatrix} \\
&= \begin{bmatrix}
1 - \frac{\ell^2}{2f^2} & 2\ell \left(1 + \frac{\ell}{2f}\right) \\
-\frac{\ell}{2f^2} \left(1 - \frac{\ell}{2f}\right) & 1 - \frac{\ell^2}{2f^2}
\end{bmatrix}.
\end{align*}
\]
For periodic motion we have $\beta_x(0) = \beta_x(2\ell)$ and $\gamma_x(0) = \gamma_x(2\ell)$, while vanishing correlation $\alpha_x$ at the two planes implies that $\beta_x(0) = 1/\gamma_x(0)$.

**Maximum beta**

$$\beta_x(0) = 2 \sqrt{\frac{2f^3 + f^2\ell}{2f - \ell}} \approx 2 |f| \left(1 + \frac{\ell}{2f}\right)$$

**Minimum beta**

$$\beta_x(\ell) \approx 2 |f| \left(1 - \frac{\ell}{2f}\right)$$

**When $f \gg l$**

$$\beta_x(z) \approx \bar{\beta}_x = 2f \quad \rightarrow \quad \langle x^2 \rangle \approx 2\varepsilon_x f$$

$$\gamma_x(z) \approx \frac{2}{\beta_x} = \frac{1}{f} \quad \rightarrow \quad \langle x'^2 \rangle \approx \frac{\varepsilon_x}{f}$$

$$\alpha_x^2(z) \approx \bar{\beta}_x \bar{\gamma}_x - 1 = 1 \quad \rightarrow \quad \langle xx' \rangle \approx \pm\varepsilon_x.$$
Electron distribution in phase space

We define the distribution function $F$ so that

$$N_e F(\Delta t, \Delta \gamma, x, x'; z) \, dx \, dx' \, d(\Delta t) \, d(\Delta \gamma)$$

is the number of electrons per unit phase space volume.

Since the number of electrons is an invariant function of $z$, distribution function satisfies **Liouville theorem**

$$\frac{d}{dz} F = \left[ \frac{\partial}{\partial z} + (\Delta t)' \frac{\partial}{\partial \Delta t} + (\Delta \gamma)' \frac{\partial}{\partial \Delta \gamma} + x' \cdot \frac{\partial}{\partial x} + x'' \cdot \frac{\partial}{\partial x'} \right] F = 0$$

**equations of motion**
Gaussian beam distribution

- Represent the ensemble of electrons with a continuous distribution function (e.g., Gaussian in $x$ and $x'$)

\[
F(x, x'; z) = \frac{1}{2\pi \epsilon_x} \exp\left\{ -\frac{1}{2\epsilon_x} \left[ \gamma_x(z)x^2 + \beta_x(z)x'^2 + 2\alpha_x(z)xx' \right] \right\}
\]

- For free space propagation

\[
F(x, x'; z) = \frac{1}{2\pi \epsilon_x} \exp\left[ -\frac{(x - x'z)^2}{2\beta_x^* \epsilon_x} - \frac{x'^2}{2\epsilon_x / \beta_x^*} \right]
\]

- Distribution in physical space can be obtained by integrating $F$ over the angle

\[
\int dx' F(x, x'; z) = \frac{\exp\left[ -\frac{x^2}{2\sigma_x^* (1 + z^2 / \beta_x^2)} \right]}{\sqrt{2\pi} \sigma_x^* \sqrt{1 + z^2 / \beta_x^2}}
\]
Photon beams

- Wave equation and paraxial approximation
- Radiation diffraction and emittance
- Transverse and temporal coherence
- Brightness and diffraction limit
- Radiation intensity and bunching
- Bright accelerator based photon sources
Photon wavelength and energy

Photon energy

- See smaller features
- Write smaller patterns
- Elemental and chemical sensitivity

\[ \hbar \omega \cdot \lambda = hc = 1239.842 \text{ eV nm} \]
**Opportunities for Tunable Source of Radiation**

- **Single pass FELs (SASE or seeded)**
- **4th-generation LS**
- **3rd-generation LS**
- **Dye lasers and their extensions**
- **IR FELs**
- **Microwave Sources**
- **Terahertz**

Various accelerator and non-acc. sources

FEL oscillators (High-average power)

Synchrotron radiation

Undulator radiation
Radiation diffraction

- Wave propagation in free space

\[
\left[ \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2} + k^2 \right] E_\omega(x; z) = 0, \quad k = \frac{\omega}{c} = \frac{2\pi}{\lambda}
\]

- Angular representation

\[
\mathcal{E}_\omega(\phi; z) = \frac{1}{\lambda^2} \int dx \ e^{-ik\phi \cdot x} E_\omega(x; z)
\]

\[
E_\omega(x; z) = \int d\phi \ e^{ik\phi \cdot x} \mathcal{E}_\omega(\phi; z).
\]

- General solution

\[
E_\omega(x; z) = \int d\phi \ \exp \left[ ik(\phi \cdot x \pm z \sqrt{1 - \phi^2}) \right] \mathcal{E}_\omega(\phi; 0)
\]

- Paraxial approximation (\(\phi^2 \ll 1\))

\[
\mathcal{E}_\omega(\phi; z) = e^{ik(1 - \phi^2 / 2)z} \mathcal{E}_\omega(\phi; 0)
\]
Gaussian beam and radiation emittance

Gaussian fundamental mode at waist $z=0$

$$E(x; 0) = E_0 \exp\left(-\frac{x^2}{4\sigma_r^2}\right)$$

$$\mathcal{E}(\phi; 0) = \mathcal{E}_0 \exp\left(-\frac{\phi^2}{4\sigma_{r'}^2}\right)$$

At arbitrary $z$

$$E(x; z) = \frac{E_0}{\sqrt{1 + i\sigma_{r'}z/\sigma_r}} \exp\left[ -\frac{x^2}{4\sigma_r^2(1 + i\sigma_{r'}z/\sigma_r)} \right]$$

$$= \frac{E_0}{{\left(1 + z^2/Z_R^2\right)}^{1/4}} \exp\left[ -\frac{x^2(1 - iz/Z_R)}{4\sigma_r^2(1 + z^2/Z_R^2)} - \frac{i}{2} \tan^{-1}\left(\frac{z}{Z_R}\right) \right]$$

$$\sigma_r(z) = \sqrt{\frac{\lambda}{4\pi}} \left(Z_R + \frac{z^2}{Z_R^2}\right)$$

Analogous with electron beam

$$\mathcal{E}_x \leftrightarrow \frac{\lambda}{4\pi}, \quad \beta^*_x \leftrightarrow Z_R.$$
What is coherence?

Complex degree of coherence

\[
\gamma(x_1, x_2, \tau) = \frac{\langle E(x_1, t) E^*(x_2, t + \tau) \rangle}{\sqrt{\langle |E(x_1, t)|^2 \rangle \langle |E(x_2, t + \tau)|^2 \rangle}}
\]

\(\gamma(x_1, x_2, 0)\) describes the transverse coherence, 
\(\gamma(0, 0, \tau)\) characterizes the temporal coherence.
Transverse (Spatial) Coherence

Transverse coherence can be measured via the interference pattern in Young's double slit experiment.

Near the center of screen, fringe visibility is described by $\gamma(x_1, x_2, 0)$.

Degree of transverse coherence (coherence fraction):

$$\zeta = \frac{\int \int |\gamma(x_1, x_2, 0)|^2 I(x_1) I(x_2) \, dx_1 \, dx_2}{\int I(x_1) \, dx_1 \int I(x_2) \, dx_2}$$
Phase space criteria for transverse coherence

- Initial phase space area $4\pi R >> \lambda$
- Final phase space area $4Ra/D \lesssim \lambda/2$
- Coherent flux is reduced by $M_T$
- This criteria from physical optics argument
**Temporal (Longitudinal) Coherence**

Coherence time is determined by measuring the path length difference over which fringes can be observed in a Michelson interferometer.

\[ \tau_c = \int_{-\infty}^{\infty} d\tau |\gamma(\tau)|^2 \]

- Temporal coherence function and the radiation spectrum forms a Fourier pair.

\[ \gamma(\tau) = \frac{\int_{-\infty}^{\infty} d\omega |E(\omega)|^2 e^{-i\omega \tau}}{\int_{-\infty}^{\infty} d\omega |E(\omega)|^2} \]

- For a Gaussian radiation spectrum,

\[ \tau_c = \frac{\sqrt{\pi}}{\sigma_\omega} \]
Define a coherence length $\ell_{\text{coh}}$ as the distance of propagation over which radiation of spectral width $\Delta\lambda$ becomes $180^\circ$ out of phase. For a wavelength $\lambda$ propagating through $N$ cycles

$$\ell_{\text{coh}} = N\lambda$$

and for a wavelength $\lambda + \Delta\lambda$, a half cycle less ($N - \frac{1}{2}$)

$$\ell_{\text{coh}} = (N - \frac{1}{2}) (\lambda + \Delta\lambda)$$

Equating the two

$$N = \frac{\lambda}{2\Delta\lambda}$$

so that

$$\ell_{\text{coh}} = \frac{\lambda^2}{2\Delta\lambda}$$
Chaotic light

- Radiation from many random emitters (Sun, SR, SASE FEL)

\[ E(t) = \sum_{j=1}^{N_e} E_0(t - t_j) = e_0 \sum_{j=1}^{N_e} \exp \left[ -\frac{(t - t_j)^2}{4\sigma_r^2} - i\omega_1(t - t_j) \right] \]

10 of the \( N_e = 100 \) randomly spaced pulses

Correlation function and coherence time

\[ C(\tau) \equiv \frac{\langle \int dt \, E(t)E^*(t + \tau) \rangle}{\langle \int dt \, |E(t)|^2 \rangle} \]

\[ t_{\text{coh}} \equiv \int dt \, |C(\tau)|^2 \]
Temporal mode and fluctuation

- Number of regular temporal regions is \# of coherent modes

\[ M_L \approx \frac{T}{t_{coh}} = \frac{T}{2\sqrt{2\pi}\sigma_T} \approx \frac{T}{5\sigma_T}. \]

- Intensity fluctuation

\[ \frac{\Delta W}{W} = \frac{1}{\sqrt{M_L}}. \]

- Same numbers of mode in frequency domain

\[ E_\omega = \frac{e_0\sigma_T}{\sqrt{\pi}} \sum_{j=1}^{N_e} \exp \left[ -\frac{(\omega - \omega_1)^2}{4\sigma_\omega^2} + i\omega t_j \right]. \]

- Fourier limit, minimum longitudinal phase space

\[ c\sigma_T \cdot \frac{\sigma_\omega}{\omega_1} = \frac{\lambda_1}{4\pi}, \]

- Longitudinal phase space is \( M_L \) larger than Fourier limit

- Total \# of modes

\[ M = M_L M_T^2. \]
Due to resonant condition, light overtakes e-beam by one radiation wavelength $\lambda_1$ per undulator period

Interaction length = undulator length

Slippage length = $\lambda_1 \times$ undulator period
(e.g., 100 m LCLS undulator has slippage length 1.5 fs, much less than 100-fs e-bunch length)

Each part of optical pulse is amplified by those electrons within a slippage length (an FEL slice)

Only slices with good beam qualities (emittance, current, energy spread) can lase
Light Bulb vs. Laser

Radiation emitted from light bulb is chaotic.

Pinhole can be used to obtain spatial coherence.

Monochromator can be used to obtain temporal coherence.

Pinhole and Monochromator can be combined for coherence.

Laser light is spatially and temporally coherent.

A. Schawlow (Nobel prize on laser spectroscopy), Scientific Americans, 1968
Brightness

$$B = \frac{\text{Photons in unit spectral range in unit time}}{(\text{source size } \times \text{divergence})^2}$$

Units: photons/s/mm²/mrad²/0.1%BW
Brightness via Wigner Function

Spectral brightness defined via Wigner function, which is Fourier transformation of the transverse correlation function (K.J. Kim, 1986).

\[
B(x, \phi; z) = \frac{d\omega \omega^2 \varepsilon_0}{\hbar \omega \pi c T} \int d\xi e^{ik\xi \cdot \phi} \left< E(x + \frac{1}{2} \xi; z) E^* (x - \frac{1}{2} \xi; z) \right>
\]

Brightness is conserved in a perfect optical system: cannot increase brightness once the source is born.

Brightness convolution theorem

\[
B(x, \phi, z) = N_e \int dx_j dx'_j B_j (x - x_j, \phi - x'_j, z) f(x_j, x'_j, z)
\]

single electron rad. brightness  electron distribution function
Radiation from many electrons

Such a beam can be described by the convolution of the coherent Gaussian beam with the electron distribution in phase space.

Same formula as previous slide except $\sigma_r \rightarrow \Sigma_r$, $\sigma_{r'} \rightarrow \Sigma_{r'}$.

\[ \Sigma_x = \sqrt{\sigma_x^2 + \sigma_r^2} \quad \Sigma_{x'} = \sqrt{\sigma_{x'}^2 + \sigma_{r'}^2}. \]

When electron beam emittance $\gg \frac{\lambda}{4\pi}$

\[ \Sigma_x \Sigma_{x'} \gg \frac{\lambda}{4\pi} \]

# of transverse modes

\[ M_T = \frac{\Sigma_x \Sigma_{x'}}{\lambda/4\pi} = \frac{\varepsilon_x}{\varepsilon_r} \]
State-of-art storage rings have pulse duration $\sim 10 \text{ ps}$, emittance $\sim 1 \text{ nm}$.

Diffraction-limited storage rings and energy recovery linacs with emittance $\sim 10 \text{ pm}$ are under active R&D.
Perfect optical system has $d_s \theta_s = d_i \theta_i$

$\theta_i$ is the numerical aperture of focusing system

Reducing pinhole size until $d_s \theta_s \sim \lambda/2$

since $d_i \sim \lambda/(2\theta_i)$ reaches diffraction limit.

A even smaller pinhole does not reduce the image size but only hurts the photon flux

Diffraction limited source does not require a pinhole and provide the most coherent flux
Storage Ring Spectral Brightness

Brightness Envelopes (4-5 m IDs)

- Diffraction-limited 2-9 GeV rings
- Upgraded 6-7 GeV rings
- New 3 GeV rings
- Existing 2-8 GeV rings

average brightness (ph/s/mm²/m².0.1%BW)

photon energy (keV)

B. Hettel
Radiation intensity

- What if emitters are not random in time

\[
\langle |E(\omega)|^2 \rangle = |E^0_\omega|^2 \left\langle \left\| \sum_{j=1}^{N_e} e^{i\omega t_j} \right\|^2 \right\rangle
\]

\[
\left\langle \left\| \sum_{j=1}^{N_e} e^{i\omega t_j} \right\|^2 \right\rangle = N_e + \left\langle \sum_{j \neq k}^{N_e} e^{i\omega(t_j-t_k)} \right\rangle
\]

\[
\left\langle \left\| \sum_{j \neq k}^{N_e} e^{i\omega(t_j-t_k)} \right\|^2 \right\rangle = N_e(N_e-1) \left| \int dt f(t) e^{i\omega t} \right|^2
\]

- For an electron bunch with rms bunch length \( \sigma_e \)

\[
f(t) = \frac{1}{\sqrt{2\pi}\sigma_e} \exp \left( -\frac{t^2}{2\sigma^2_e} \right)
\]

\[
\left\langle |E(\omega)|^2 \right\rangle = N_e |E^0_\omega|^2 \left[ 1 + (N_e - 1)e^{-\omega^2\sigma_e^2} \right]
\]

- When intensity from many electrons add incoherently (~\( N_e \))
Bunching and coherent radiation

- If the bunch length is shorter than the radiation wavelength
  \[(N_e - 1)e^{-\omega^2\sigma_e^2} \geq 1\]

- Radiation intensity from many electrons add coherently \((\sim N_e^2)\)

- Another way to produce bunching from a relatively long bunch is through so-called microbunching

\[
\langle |E(\omega)|^2 \rangle = N_e |E_\omega|^2 \left(1 + (N_e - 1)|f(\omega)|^2\right)
\]
Resonant interaction of electrons with EM radiation in an undulator

Coherent radiation intensity \( \propto N^2 \) due to beam microbunching

\( N: \# \text{ of } e^- \text{ involved } \sim 10^6 \text{ to } 10^9 \)

At x-ray wavelengths, use **Self-Amplified Spontaneous Emission*** (a wonderful instability!) to reach high peak power

* Kondradenko, Saldin, Part. Accel., 1980
* Bonifacio, Pellegrini, Narducci, Opt. Com., 1984

^ J. Madey, J. Appl. Phys., 1971
Evolution of X-ray Light Sources

GE synchrotron (1946) opened a new era of accelerator-based light sources.

These light sources have evolved rapidly over four generations.

- The first three generations are based on synchrotron radiation.
- The fourth generation light source is a game-changer based on FELs.

The dramatic improvement of brightness and coherence over 60 years easily outran Moore’s law.
X-Ray Holography: Coherence Wanted

Lensless imaging of magnetic nanostructures by x-ray holography

Future Role of FELs and Advanced Sources

Ordered Structures
Equilibrium Phenomena

Disordered Structures
Nonequilibrium Phenomena
Transient States

Era of Crystalline Matter
Conventional X-ray Probes

Era of Disordered Matter
Coherent X-ray Probes

1900 2000 future

H. Dosch (DESY)
Summary

Despite spectacular successes in synchrotron radiation and FELs, the quest for brightness and coherence continues, with no sign of slowing down.

Future light source development includes diffraction-limited light sources, high-peak and average power FELs, compact coherent sources and many more possibilities.

I hope you enjoy this summer school and this exciting field of research.