Measuring high brightness beams

C. Limborg-Deprey
Introduction

Measuring e-beam properties

emittance, charge, monochromaticity, pulse duration …

\[
B = \frac{Q}{\varepsilon_{n,x} \varepsilon_{n,y} \varepsilon_{n,z}}
\]

\[
B = \frac{Q}{\varepsilon_{n,x}^2 \sigma \sigma_E}
\]

Standard measurement techniques for tuning FELs
Some measurements of ultimate beam dimensions for colliders and e-diffraction sources
Synopsis

- Emittance definition
- Transverse emittance measurements
- Beam size measurements
  - Destructive
  - Non-Destructive
- Bunch lengths
  - Non-Destructive
  - Destructive
- Longitudinal Phase space
6D-emittance

Evolution of a population of particles (~$10^9$ for 160 pC) in time

3D-space coordinates ($x$, $y$, $z$) and their canonical (Hamilton) conjugates ($p_x, p_y, p_z$)

6D phase space of non-interacting particles, in a conservative dynamical system is invariant

$\to$ “Liouville theorem”

Volume $V = \int \psi(x, p_x, y, p_y, y, z, p_z) \, dx \, dp_x \, dy \, dp_y \, dz \, dp_z$ conserved

$V$ referred to as emittance

6D-emittance – more practical choice of coordinates

Particles travel along $z$ with relativistic velocity $v = \beta c$ and $p_z = \beta \gamma m_o c$

then $p_x = v_x \gamma m_o = \frac{dx}{dz} \gamma m_o = x' \beta c \gamma m_o$

$p_x \to \frac{p_x}{p_{zo}} = x'$

$p_y \to \frac{p_y}{p_{zo}} = y'$

$z \to \frac{(z - z_o)}{\beta c} = \tau$

$p_z \to \delta = \frac{p_z - p_{zo}}{p_{zo}}$ or $\delta = \frac{E - E_o}{E_o}$

$(x, x')$ are the trace-space coordinates (not conjugate quantities)

$(x, p_x, y, p_y, y, z, p_z) \to (x, x', y, y', \tau, \delta)$ (measured quantities)

subtleties of emittance conservation are discussed in K.Floettmann, “some basic features of the beam emittance” PRST-AB Vol.3, 034202 (2003)
Geometric emittance / Normalized emittance

\[ \varepsilon = \int f(x, x', y, y', \tau, \delta) \, dx \, dx' \, dy \, dy' \, d\tau \, d\delta \]

- \( \varepsilon \) geometric emittance
- \( \varepsilon \) conserved in the absence of acceleration
  
  \( (\varepsilon \text{ decreases with } 1/\gamma^3 \text{ when beam accelerated}) \)

we introduce \( \varepsilon_n = \gamma^3 \varepsilon \), normalized emittance \( \varepsilon_n \)

\( \varepsilon_n \) conserved in the presence of acceleration

-----------------------------  Possible sources of emittance growth -----------------------------

Non-linear space charge forces
Non-linear forces from electromagnetic components
Synchrotron radiation emission
Emittance - Practical evaluation

2D emittance: area occupied by particles represented in 2D “trace space”

What is the “area” for points?
→ arbitrarily define a contour

Typical distributions are gaussians, or if not, can be evaluated by their rms value, $< u^2 >, < u'^2 >, < uu' >$
Emittance - Practical evaluation

- Determinant of the matrix of second order moments

\[
\begin{vmatrix}
< u^2 > & < uu' > \\
< uu' > & < u'^2 >
\end{vmatrix}^{1/2}
\]

\[\varepsilon = \sqrt{< u^2 > < u'^2 > - < uu' >}\]

Corresponds to \( \varepsilon = \frac{A}{\pi} = \sigma_M \sigma_m \)

- Another definition: “Lapostolle”

\[\bar{\varepsilon} = 4 \sqrt{< u^2 > < u'^2 > - < uu' >}\]

Corresponds to \( \frac{A^*}{\pi} = 2 \sigma_M \ 2 \sigma_m \)
Emittance

Define Twiss parameters $\beta, \alpha, \gamma$ by

\[
\begin{align*}
< u^2 > &= \beta \varepsilon \\
< u'^2 > &= \gamma \varepsilon \\
< uu' > &= -\alpha \varepsilon
\end{align*}
\]

$\beta, \alpha, \gamma$ define the orientation of the ellipse

$\pi \varepsilon$ defines the area
Another approach to emittance

Particles \((u,u')\) subject to restoring forces \(K(s)u\)

\[ u'' + K(s)u = 0 \quad \text{Hill's equation} \]

Solution: \(u(s) = \sqrt{\varepsilon \beta(s)} \sin(\psi(s) + \phi_o)\)

with \(\psi(s) = \int_{s_0}^{s} \frac{ds'}{\beta(s')}\)

\(\varepsilon\) and \(\phi_o\) are constants of integration

Defining: \(\alpha(s) = -\frac{1}{2} \beta'(s)\) and \(\gamma(s) = \frac{1+\alpha^2(s)}{\beta^2(s)}\)

\(\gamma(s)u^2(s) + 2\alpha(s)u(s)u'(s) + \beta(s)u'^2(s) = \text{constant} \quad \text{(Courant-Snyder Invariant)} = \varepsilon\)

Also equation of an ellipse

Explains why choice of an ellipse is more than a mathematical convenience
Measuring emittance

\[ \varepsilon = \sqrt{< u^2 > < u'^2 > - < uu' >} \]

Beam size  
Beam divergence  
Correlation Position-angle

One method gives the 3 parameters in one shot: “Pepper pot”
but

destructive
only for low energy beam (E < 10 MeV)
Transverse emittance, low energy
Destructive
Single Shot

Pepper pot

Multislit in high Z material

Direct reading of trace space

\[ u'_m = x'_m = \frac{u_m - x_m}{L} = \frac{u_m - m d}{L} \]

\[ \sigma'^2_m = \sigma^2_{u,m} - \left( \frac{a}{4} \right)^2 \]

\[ < x^2 > = \frac{\sum I_m x_m^2}{\sum I_m} \]

\[ < x'^2 > = \frac{\sum I_m (x'_{m,0}^2 + \sigma'^2_m)}{\sum I_m} \]

\[ < xx' > = \frac{\sum I_m x_{m,0} x'_{m,0}}{\sum I_m} \]

With \( I_m \) detected intensity for \( m^{th} \) beamlet

Pepper pot for thermal emittance

Electron beam on YAG, with pepper pot in, Solenoid scan

C.P. Hauri, PRL 104, 234802 (2010)
Pepper pot based emittance oscillation

Emittance-meter
Variation of emittance along z after gun (at 5 MeV)

2mm thick W
7 slits of ~ 50 µm at 500 µm
Resolution ~ 100 µrad

D. Alesini et al., “Experimental results with the SPARC emittance-meter” MOOAAB02, PAC07
Photofield emission from tips: Nb/ Nb3Sn

No explosive electron emission erosion even if close to 1GW/cm^2 limit
Huge QE ~ 0.9 % (X 100)
εₙ = 0.6 mm-mrad at 2500pC
Brightness better by ~ 100 than conventional PC guns

Pepper pot

- **Difficult at high energy**
  
  scattering stopping length longer with energy:

  \[ L_s[cm] = \frac{E}{dE/dx} \cong \frac{E[MeV]}{1.5 \rho[g.cm^{-3}]} \]

  
  E > 10 MeV, \( L_s \sim 5 \text{ mm} \)

- **Unsuitable for extremely low emittance**
  
  slit thickness condition: \( \sigma_{x'} < \frac{a}{4w} \)

  minimum emittance \( \propto a \) (slit width) but S/N decreases

- **Space Charge correction**: low energy: analysis still often requires space charge calculation

<table>
<thead>
<tr>
<th>W</th>
<th>Ta</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.6</td>
<td>16.7</td>
<td>8.96</td>
</tr>
</tbody>
</table>

Grid for low emittance

Measuring extremely low emittance $<0.1 \text{ mm-mrad}$

Grid technique similar to pepper pot,
  multi knife edge measurement with info. on correlations
  electrons hitting Cu bars scattered at wide angle

huge magnification $M=12.8$ (from $L=2 \text{ m}$)

\[ M \geq L \sigma_m' \]

$a$ : bar width $= 32 \mu\text{m}$

$L$ : distance grid to screen $=2.1 \text{ m}$

$\sigma_m'$ : divergence of beamlet $\sim 40 \mu\text{rad}$

30nm measured with 0.1 pC

R.Li et PRST-AB 15, 090702 (2012)
Grid for low emittance

Studying cigar shape:
linear space charge

R.Li et PRST-AB 15, 090702 (2012)
C.Limborg-Deprey, SLAC, SSSEPB Ji
**“Quad” scan**

Quadrupole/solenoid scan

\[
\begin{pmatrix}
 x \\
 x'
\end{pmatrix}
= \begin{pmatrix}
 1 & L \\
 0 & 1
\end{pmatrix}
\begin{pmatrix}
 1 & -\frac{1}{f} \\
 0 & 1
\end{pmatrix}
\begin{pmatrix}
 x_o \\
 x'_o
\end{pmatrix}
= R
\begin{pmatrix}
 x_o \\
 x'_o
\end{pmatrix}
\]

\[
\begin{cases}
 x = \left(1 - \frac{L}{f}\right)x_o + Lx_o \\
 x' = -\frac{x_o}{f} + x'_o
\end{cases}
\]

\[
\langle x^2 \rangle = \left(1 - \frac{2L}{f} + \frac{L^2}{f^2}\right)\langle x_o^2 \rangle + 2L \left(1 - \frac{L}{f}\right)\langle x_o x'_o \rangle + L^2\langle x'_o^2 \rangle
\]

\[
\langle x^2 \rangle = \frac{a}{f^2} + \frac{b}{f} + c
\]

Solve for a, b, c to determine \(\langle x_o^2 \rangle, \langle x_o x'_o \rangle, \langle x'_o^2 \rangle\)

\[
\varepsilon = \sqrt{\langle x_o^2 \rangle \langle x'_o^2 \rangle - \langle x_o x'_o \rangle}, \quad \alpha = -\frac{\langle x_o x'_o \rangle}{\varepsilon}, \quad \beta = \frac{\langle x_o^2 \rangle}{\varepsilon}, \quad \gamma = \frac{\langle x'_o^2 \rangle}{\varepsilon}
\]
"Quad" scan

\[
\begin{pmatrix}
\sigma_{x_1}^2 \\
\sigma_{x_2}^2 \\
\vdots \\
\sigma_{x_n}^2
\end{pmatrix} =
\begin{pmatrix}
R_{11,1}^2 & 2R_{11,1}R_{12,1} & R_{12,1}^2 \\
\vdots & \vdots & \vdots \\
R_{11,n}^2 & 2R_{11,n}R_{12,n} & R_{12,n}^2
\end{pmatrix}
\begin{pmatrix}
\beta_o \varepsilon \\
-\alpha_o \varepsilon \\
\gamma_o \varepsilon
\end{pmatrix}
\]
also written \( \Sigma = BO \)

Solve for \( O \) by least square fit

Minimization of \( \chi^2 = \sum_i \frac{1}{\sigma_{x_i}^2} \left( \sigma_{x_i}^2 - \sum_j B_{ij} O_j \right)^2 \)

Error analysis

Calculate covariant matrix \( T = (\hat{B}^T \hat{B})^{-1} \)

where \( \hat{B}_{ij} = \frac{B_{ij}}{\sigma_{x_i}^2} \)

Solution is \( O = T \hat{B}^T \hat{\Sigma} \) where \( \hat{B}_{ij} = \frac{B_{ij}}{\sigma_{x_i}^2} \) and \( \hat{\Sigma}_i = \sigma_{x_i}^2 \)

Error \( \sigma_{O,i} = \sqrt{T_{ii}} \)

Reference "F.Zimmermann, M.Minty, “ book ref. H.Loos , private communications
“Quad” scan

- Choice of quadrupole settings

At each data point, plot lines parametrized in $x'_i$

\[
(u) = R \left( \sigma_{x_i} \right)
\]

Optimum location of data points

over-constrain (N points $> 3$)

at least 3 separated by more than 45 degrees

typically ~ 7 points, covering more than 90 degrees

- Convenient representation in normalized/rotated phase space

\[
\left( \frac{x}{\sqrt{\beta}}, \frac{\alpha x + \beta x'}{\sqrt{\beta}} \right) \text{ where } (\alpha, \beta) \text{ Twiss design parameters}
\]
“Quad” scan

- Low energy or high density strong space charge,

\[ R \text{ to include space charge defocusing term (beam size dependent)} \]

\[ R_{sc} (\Delta s) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\Delta s/ f_{sc,x} & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & \Delta s/ f_{sc,y} & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \Delta s/ f_{sc,z} & 1
\end{pmatrix} \]

\[ \frac{1}{f_{sc,x}} = K/2 \left[ \frac{1}{5} X^2 \right]^{3/2} \left[ R_d \left[ \frac{Y^2}{X^2}, \frac{Z^2}{X^2}, 1 \right] \right] \]

\[ \frac{1}{f_{sc,y}} = K/2 \left[ \frac{1}{5} Y^2 \right]^{3/2} \left[ R_d \left[ \frac{Z^2}{Y^2}, \frac{X^2}{Y^2}, 1 \right] \right] \]

\[ \frac{1}{f_{sc,z}} = K/2 \left[ \frac{1}{5} Z^2 \right]^{3/2} \left[ R_d \left[ \frac{X^2}{Z^2}, \frac{Y^2}{Z^2}, 1 \right] \right] \]

\[ K = \frac{qN}{2\pi\varepsilon_0 \gamma^3 \beta^2 mc^2} \]

Use iterative methods

C.Limborg, “A Modified Quad Scan technique for space charge dominated beams”, PAC03, http://accelconf.web.cern.ch/AccelConf/p03/PAPERS/WPPG033.PDF
Slice emittance

Transverse deflector

Projects time into space

Quadrupole x-focusing
Slice emittance

Destructive Multi Shot

Slice emittance
Thermal emittance

- Thermal emittance measurement from slice emittance as a function of laser spot size (at low charge ~20pC)
- LCLS gun: copper (robustness, long lifetime)
- 0.9 mm-mrad per mm rms from slice emittance measurement

\[
\gamma_x (\mu m) = 0.91 \pm 0.01 \mu m/mm \\
\text{theory} = 0.50 \mu m/mm
\]
Thermal emittance

Thermal emittance measurement from photoinjector

\[ \varepsilon_n \sim 1.1 \pm 0.2 \text{ mm.mrad/mm} \]

\[ \text{QE} \sim 9 \times 10^{-5} \]

\[ Q = 2 \text{ pC} \text{ at } 0.27 \text{ mm laser; Jaguar, RF Phase = 95 deg; Ener. Spread; Sol=95A; 6.85 MeV; } \beta y = 14.36; \text{ RF power = 19 MW; shutter=1.5s; } \varepsilon_{\text{trans}} = 0.486 \text{ mrad; all lines: 100% charg} \]

\[ 49 \text{ MV/m} \]

Cu_1     Removed cathode

\[ 0.42 \text{ pC/mm}^2 \text{ at RF Phase = 9 deg; } 7.1 \text{ MeV/c; } \beta y = 13.9; \text{ Solenoid = 175.9mT; No quads; 21 Shots; Pepperpot 28.05.10} \]

\[ \text{QE} \sim 1 \times 10^{-5} \]

Diamond milled Cu_1

\[ \varepsilon_n \sim 0.53 \text{ mm.mrad/mm} \]

28.05.10

Laser Spot Size (mm rms)

Emitance (mm.mrad)

Laser Spot Size (mm rms)

Emitance (mm.mrad)
Thermal emittance

Quantum Efficiency

Laser Wavelength (nm)

Laser Wavelength (nm)

Photon Energy (eV)

$\text{QE} \sim (h\nu - \Phi_{\text{eff}})^2$

$\varepsilon_{\text{nx}} \sim \sqrt{h\nu - \Phi_{\text{eff}}}$

C.P. Hauri, PRL 104, 234802 (2010)
Emittance, Temperature, Coherence Length

\[ \varepsilon_n = \sigma_x \frac{\sigma_{px}}{mc} \] for e-source

\[ \varepsilon_n = \sigma_x \sqrt{\frac{k_b T}{mc^2}} \] transverse velocities expressed in terms of temperature

Typical numbers for photo-injectors (metallic or semi-conductor):

\[ \varepsilon_n = 0.9 \text{ mm} - \text{mrad per mm rms} \]

\[ T = 4809 \text{ K} , \ k_b T = 0.41 \text{ eV} \]

FELs, linear colliders: small beam size collimated beam

“geometric emittance” \( \varepsilon = \frac{\varepsilon_n}{\gamma} \) small at high \( \gamma \) and small \( \varepsilon_n \)

For electron diffraction community, coherence length

\[ L_c = \frac{\lambda}{2\pi \sigma_\theta} = \frac{h}{mc} \frac{\sigma_x}{\varepsilon_{n,x}} \]

\( L_c \) needs to be many lattice spacing

\[
\begin{align*}
\text{example:} & \quad \varepsilon_n = 0.02 \text{ mm} - \text{mrad} \\
& \quad \sigma_x = 0.2 \text{ mm}
\end{align*}
\]

\[ L_c = 4 \text{ nm} \]

Quality of source measured in \( C_\perp = \frac{L_\perp}{\sigma_x} \)
Cold Electron source

N \sim 10^8 \text{ Rb atoms}
R = 1\text{mm}
T = 230 \mu \text{K}

Electron temperature

\begin{align*}
\text{Rb}^+ & \quad \uparrow E_{\text{exc}} \\
< 480 \text{ nm} & \quad \text{Ionization} \\
5 \text{ P}_{3/2} & \quad 780 \text{ nm} \quad \text{Excitation} \\
5 \text{ S}_{1/2} & \\
\rightarrow T_e \approx 10 \text{ K}
\end{align*}

Magneto-Optical Trap (MOT)  
Ultracold Plasma

Killian et al., PRL 83, 4776 (1999)

Courtesy O.J. Luiten
Cold Electron source

Electron temperature

\[ kT_e \approx \frac{\hbar}{\tau} \]

Conventional photo and field emission fields

\[ T_e \approx 5000 \text{ K (0.5 eV)} \rightarrow 20 \text{K} \]

\[ \varepsilon_n = \sigma_x \sqrt{\frac{k_b T}{mc^2}} \]

\[ T \equiv \frac{\sigma_p^2}{mk_b} \]

\[ T = 5000 \text{ K for 0.9 mm-mrad per rms mm} \]

\[ T = 1500 \text{ K for 0.5 mm-mmrad per rms mm} \]

Courtesy O.J.Luiten
Solenoid scan for extremely small emittance

\[ \sigma_{e_1} \sim 20 \, \mu m \] controlled by ionization laser

Energy measured with Time of Flight from ionization laser (photodiode) to screen
0.4 fC measured on Faraday Cup (~ 3000 particles)

MCP-phosphor screen (photo-multiplication) for enhancing signal for single electron detection

Pinhole for resolution determination \( \sim 100 \, \mu m \)

Courtesy O.J.Luiten

Engelen et al. Nature Communications “High-coherence electron bunches produced by femtosecond”
Engelen et al “Effective temperature of an ultracold electron source based on near-threshold Photoionization”
Solenoid scan for an extremely small emittance

\[ \varepsilon = 1.4 \, nm \] for \( Q = 0.5 \, fC \) (\( \sim 3000 \) particles)

Equivalent to 70 nm-rad per mm rms

\sim 10 \text{ times smaller than RF photoinjectors}

So 100 \text{ times larger brightness}

Main conclusion

fs-ionization still allows to reach 20K thermal emittance

Engelen et.al Nature Communications “High-coherence electron bunches produced by femtosecond”
Engelen et al “Effective temperature of an ultracold electron source based on near-threshold Photoionization” Ultramicroscopy
Spatial projection: choice of “screens”


Screens

ZnS phosphor screens
  - small dynamic range, easily saturated
  - grain size ~ 20μm
  - response time

YAG screens:
  - good yield
  - thickness
  - response time


YAG combined with MCP for low electron density

OTR screen

Wire scanner

Ultimate wire
OTR (Optical Transition Radiation)

• Emitted when a relativistic electron passes boundary between materials of different electrical properties

• OTR emission from a single electron
  • Travelling with relativistic velocity $\beta \approx 1 - \frac{1}{2\gamma^2}$
  • Hitting a foil with reflectivity $R(\omega)$ foil
  • Typically at 45 degrees w.r.t to beam direction

$$\frac{d^2N}{d\omega d\Omega} = \frac{\alpha}{4\pi^2} \frac{R(\omega)}{\omega} \frac{\beta^2 \sin^2(\theta)}{(1-\beta \cos(\theta))^2}$$

$\theta > 150 \text{ mrad}, 60\% \text{ of light}$
OTR far-field imaging

Can we do far-field imaging and obtain the e-beam divergence?

Deconvolution of e-beam divergence with single e angular distribution

Resolution \( \chi' \sim \frac{0.15}{\gamma} \)

(too poor for high brightness beams)

but accurate energy measurement

OTR interferometry
Back TR from 1\textsuperscript{st} foil
Distance between 2 foils > \( \lambda \gamma^2 \)

Resolution \( \chi' \sim \frac{0.075}{\gamma} \)

Simulations P.Piot, 38 MeV

R.Fiorito OTR-ODR interferometry \( \chi' \sim \frac{0.02}{\gamma} \)
• Calculate $N_{\text{photons}}$
  • Spectral reflectivity of foil (Al, Ti, Be, Si)
  • Spectral sensitivity of imaging system (lens, CCD)

$$S(\lambda_i, \lambda_f) = \int_{\omega_i}^{\omega_f} \frac{R(\omega)}{\omega} S(\omega) d\omega$$

$$\int_{\omega_i}^{\omega_f} \frac{R(\omega)}{\omega} d\omega = [\log \omega]_{\lambda_i=400nm}^{\lambda_f=700nm} = 0.56$$

Quantum Efficiency from Sony ICX445
Imaging using OTR light

Beam size: “point-to-point imaging”
Point Spread Function (PSF): single e emission (Green’s) through imaging system

$$PSF * \rho(x, y)$$ (for incoherent emission)

Example:
- 50 µm, 1:1 imaging, 100pC, 70 MeV
- Collection angle = 100mrad
- BW=0.7, QE = 0.5
- $$N_{electrons} \approx 2 \times 10^6$$
- Disk with 1.5 rms contains 70% of particles
- With pixel size (3.75 µm)$^2$, ~1000 e per pixel

NB: full well capacity 30000,
With 8 bits above noise => noise level at 100e
~ 10 pC
OTR (Optical Transition Radiation)

- PSF “Point Spread Function”: response of imaging system to a point source

- Incoherent:
  \[ Image(O_1 + O_2) = Image(O_1) + Image(O_2) \]
  \[ PSF \ast \rho(x, y) \Rightarrow \text{intensity } \propto N_e \]

- Coherent: image formation linear in complex field
  In short \( FT^{-1}(E_s(k_x, k_y) \times \rho(k_x, k_y)) \)

\[
E(r) = \frac{q}{\pi \nu} \frac{r}{r} \alpha K_1(\alpha r) \quad \text{electric field at the transition radiation source with } \alpha = \frac{k}{\beta \gamma}
\]

\[
E_{S,N}(r) = \sum_j e^{-jk_jz_j} E_s(r - r_j)
\]

\[
|E_{S,N}(r)|^2 = N \int d^2 r' \, d\, z \, \rho(r', z) |E_{S,N}(r - r')|^2
\]

\[ + N^2 \int d^2 r' \, d\, z \, e^{-ikz} \rho(r', z) E_{S,N}(r - r')|^2
\]

- Microbunching generates phase relations for particles along bunch making the N2 component large and dominant
Coherent OTR (COTR)

Coherent emission:

OTR image show square of gradient of shape
thus the “doughnut” shape

Figure 10: Series of COTR color images taken within one minute showing varying color content from shot to shot.

Spoils measurement of rms beam size (under estimate)
Starts from shot noise => present in continuous spectrum
OTR for beam size measurement

COTR suppression:
Smearing out microbunching
Spectral separation: imaging for $\lambda < \lambda_c$ (i.e., extreme UV, in vacuum)
Spatial separation: Use scintillator screen, tilted to avoid COTR
Temporal separation: (scintillator response $\sim 70$ ns) with expensive ICCD
Use Phase retrieval algorithm (A. Marinelli) on far-field COTR
Beam size measurement despite COTR

Far Field COTR

|B (k_x, k_y)|

Phase{B (k_x, k_y)}

Reconstructed image (beam is micro-bunched)

No-microbunching Direct OTR imaging

A, Marinelli et al.
PRL 110, 094802 (2013)
Small wire intercepts beam
Bremsstrahlung radiation and scattered electrons detected on fast ion chambers with PMTs (PMT = Photo-Multiplying tubes)

- detecting Cerenkov from secondary electrons
- light from plastic scintillator fibers along chamber

Positioned at low background location

Material

- W more signal but burn more easily ~ 20-40 μm
- C less signal, but more robust can go down to few μm

for a 4 μm rms beam sizes down to 1 μm can be measured

Major issues

- break (high rep. rate and high Ipeak) 2nC in wire of 1 μm
- vibrations
- beam jitter (from magnet power supplies stability)
Wire scanner

Non-destructive beam size
Multi Shot

- Vibrations are a limitation
  - resonance studies: speed tests
  - optical measurements

H. Loos, MOIANB01 Proceedings of BIW10, Santa Fe, New Mexico, US

SLAC PUB-6015 McCormick, M. Ross
Mechanical improvements to reduce further
Compton scattering process

Collision photons with relativistic electrons

Scattered photons

- Direction: in cone with opening angle in \(1/\gamma_0\)

- Frequency: Doppler shifted
  \[ h\nu_{sc} = h\nu_o \left( \frac{\gamma_o E_o (1-\beta_o \cos(\psi))}{\gamma_o E_o (1-\beta_o \cos(\psi)) + h\nu_o (1-\cos(\psi-\theta))} \right) \approx 2\gamma_o^2 h\nu_o \text{ for } \psi \approx \frac{\pi}{2} \]

- Intensity:
  \[ N_{sc} \approx \sigma_c \langle n_o \rangle N_{electrons} D \quad \text{with } \langle n_o \rangle = \frac{1}{h\nu_o} \frac{P}{c S} \]
  
  \( (P,S) \) : (power, spot size) of laser
  
  \( D \) : distance of interaction
  
  \( \sigma_c \) : Compton cross section \( \approx 6.6 \times 10^{-29} m^2 \)
Laser wire scanner

- measure e-beam sizes > 200nm,
- high current beams ("non-degradable" wire)
- detection of X-Ray (or $\gamma$-rays) or recoil energy (for very high $E_{\text{e-beam}}$)
- scan either laser or e-beam position
- detection requires bending magnet
- challenges
  - synchronization
  - focus
  - scan
Laser wire scanner

- Measurements SLC (pioneering experiment)

E = 45 GeV,

$N_{\text{photons}}$ at waist $\sim 8000$

Bunch size measured $\sim 1\mu$m
Focus:

Diffraction limit of a laser beam

$$\sigma_f = \frac{\lambda f}{4\pi \sigma_{laser,ini}}$$

Rayleigh length

$$z_r = \frac{4\pi \sigma_f^2}{\lambda}$$

Example: $$\sigma_f \sim 125 \text{nm}, \; z_r \sim 625 \text{ nm for } \lambda = 300\text{nm}$$

Beam size measurements limited to ~ 200nm

Applications: ERL, linear colliders

Other application: bunch length measurement

References:
A.Murohk, IPAC10 “A 10 MHZ Pulsed Laser Wire scanner for energy recovery linacs”
Beam size from laser interferometer

- measure e-beam sizes < 200nm, by beating the diffraction limit using interferences

\[
\langle N_\gamma \rangle = A + B \cos(2k_Lx + \psi)
\]

\[
N_\gamma(y_0) = \frac{N_0}{2} \left( 1 + \cos(2k_Ly_0) \cos(\theta) \exp\left[ -2(k_y \sigma_y)^2 \right] \right)
\]

\[
N_\pm = \frac{N_0}{2} \left( 1 \pm \cos(\theta) \exp\left[ -2(k_y \sigma_y)^2 \right] \right)
\]

\[
M = \frac{N_+ - N_-}{N_+ + N_-}
\]

\[
\sigma_y = \frac{d}{2\pi} \sqrt{2 \ln \left( \frac{|\cos \theta|}{M} \right)}
\]

With \( \theta = 174^\circ \), \( \sigma_y \) down to 38 nm can be measured with a 1\( \mu \)m laser

"Shintake-monitor"
Beam size from laser interferometer

- Systematics
  - Imbalance power between 2 arms
  - Variation in e-beam angle
  - Longitudinal extent of diffraction pattern
  - Spherical wavefront error
  - Coherence of laser (transverse + longitudinal)

Courtesy Jacqueline Yan, KEK

Non-destructive
Non-degradable
Multi Shot
Beam size from laser interferometer

- Shintake pioneering measurement SLAC (1994)
  
  ![FFT@SLAC](image)
  
  \[ E = 46.6 \text{ GeV} \]
  \[ \sigma_y \sim 77 \text{ nm} \]
  

- KEK measurements
  
  1.6nC
  
  1.3 GeV, \( \theta = 174^\circ \)
  
  Very reproducible over 20 minutes
  
  \( M \sim 0.306 \) ie \( \sigma_y \sim 65 \text{ nm} \)
  
  Best measurement \( \sigma_y \sim 58 \text{ nm} \)
  
  Goal = 37 nm

Courtesy Jacqueline Yan, KEK,

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Optical Diffraction Radiation Interferometry (ODRI)  
Non-destructive  
For single shot x,x’

- 0.4nC * 20 bunches at 5Hz ~ 80nC enough to damage OTR
- 2 apertures at $d < \lambda \gamma^2$ ("formation length") but with different aperture size

http://prst-ab.aps.org/pdf/PRSTAB/v14/i10/e102803

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Optical Diffraction Radiation Interferometry (ODRI)

- Calculation

Measurements

\[ \sigma_y = 81\mu m, \sigma_{y'} = 203\ \mu rad \]

http://prst-ab.aps.org/pdf/PRSTAB/v14/i10/e102803
“single point” emittance measurement

\[
\begin{align*}
\langle x^2 \rangle &= \left(1 - \frac{L}{f}\right)^2 \langle x_0^2 \rangle + 2L \left(1 - \frac{L}{f}\right) \langle x_0 x'_o \rangle + L^2 \langle x'_o^2 \rangle \\
\langle xx' \rangle &= -\frac{1}{f} \left(1 - \frac{L}{f}\right) \langle x_0^2 \rangle + \left(1 - \frac{2L}{f}\right) \langle x_0 x'_o \rangle + L \langle x'_o^2 \rangle
\end{align*}
\]

Find magnet setting \( f^* \) such that \( \frac{\partial \langle x^2 \rangle}{\partial f} = 0 \)

then correlation term \( \langle xx' \rangle = \frac{\langle x^2 \rangle}{L} \)

at \( f^* \), measure \( \langle x^2 \rangle \), and deduce \( \langle xx' \rangle \)
measure \( \langle x'_o^2 \rangle \)

\( \Rightarrow \) emittance at single point
(with 2 images: one near-field, one far-field)
BUNCH LENGTHS
Electro-Optic methods

Bunch length
Non-destructive

\( \overline{E}_r \) component of Lorentz contracted Coulomb field

Birefringence induced by \( \overline{E}_r \), \( P = \varepsilon_0 \left( \chi_e^{(0)} E + \chi_e^{(1)} E^2 + \cdots \right) \)

Pockels Kerr

Changes polarization and introduces phase retardation

As an \( E \)-field pulse mixer, casually referred to as an “ultrafast Pockels cell”

500 MeV (\( \gamma = 1000 \))
\( r = 5\text{mm} \)
\( L = 10 \mu\text{m} = 33 \text{fs} \)
Electro-Optic Sampling

1. Laser pulse much shorter than e-beam pulse used as probe
   • Samples small part of e-beam field
2. Analyzer (polarizer) to transmit polarization change to photo-diode
3. Scan delay to trace out e-beam profile

Limitation as multi-shot method:
• Shot-to-shot timing jitter bad for profiling
• (Still useful time of arrival monitor)

from Bernd Steffen, DESY
Electro-Optic spectral decoding

1. Laser pulsed stretched: long chirped laser with linear $\lambda$-$t$ correlation
2. Interaction with $E_r$ from e-beam in EO induces $t$-dep. change in polarization
3. Analyzer/spectrometer measures change in polarization as function of $\lambda$
4. Map $\lambda$ back to $t$ to deduce e-beam profile

Limitation from non-linear optical mixing:
• Resolution $t_{res}$ worsens with increasing chirp duration “window” $T_c$ as $t_{res} \propto \sqrt{T_p T_c}$

Typ. temporal resolution > 200fs
Electro-Optic temporal decoding

- Similar to before but pulse is decoded in time-domain
- Decoding setup similar to fast, optical cross-correlator

Limitations: again, gate width thickness of BBO spatial resolution

\[ I_{SHG}(x) = \int I_{probe}(t + \tau) + I_{gate}(t) \, dt \]
Temporal resolution achieved \( \sim 60 \) fs
Benchmarked with transverse deflector measurements

Electro-Optic spatially resolved

Bunch length
Single Shot

A.L Cavalieri PRL 94, 114801 (2005)

Resolution ~ 115fs rms

B. Steffen PhD manuscript,
C. Casalbuoni, PRST-AB, 11, 072802 (2008)
T. Maxwell http://www.niu.edu/physics/_pdf/academic/grad/theses/Maxwell
C. Limborg-Deprey, SLAC, SSSEPB July 24th

FIG. 3 (color). (a) Twenty consecutive single-shot electron bunch measurements. The bright band in each column is the electro-optic signal, its location indicates the time of arrival of the electron bunch with respect to the laser probe pulse, and its width corresponds to the electron bunch duration. (b) Normalized arrival time histogram of 1000 consecutive single shots.
Electro-Optic limits

Maximize signal but minimize crystal size given all the broadening contributions

TO: Transverse Optical phonons limit bandwidth
GVM: Group Velocity Mismatch
  (signal and probe slip through each other)
GVD: Group Velocity Dispersion
  (signal and probe broaden)

Organic crystal DAST, compared to ZnTe
Both crystals are 2mm from beam, and 280 pC

C.Casalbuoni, PRST-AB, 11, 072802 (2008)
H.Tomizawa, BIW 2012
3D –monitor  Tomizawa

3D bunch shape monitor (One element of 3D-BCDM) [1]

Transverse detection (2D-charge moments of bunch slice)

Simulation Parameters:
- Energy: 8 GeV
- Charge: 100 pC
- Bunch duration: 30 fs
- Detection points: 2 mm from beam axis

The signal intensity at the crystals:

\[ I = I_0 \sin^2 \left( \frac{\pi}{\lambda} n_0 \frac{r_1}{r_d} E \right) \]
Transverse deflector (TCAV)

Bunch length
Destructive
Single Shot

Δy′(z) = \( \frac{eV_o}{pc} \sin(k_{rf}z + \varphi_{rf}) \)

Δy = \( \sqrt{\beta_{OTR}\beta_{TCAV}} \sin \psi_{TCAV\rightarrow OTR} \) \( \frac{eV_o \sin(k_{rf}z + \varphi_{rf})}{pc} \)

Resolution:

\( \sigma_y^2 = \epsilon \beta_{OTR} + \beta_{OTR}\beta_{TCAV} \sigma_z^2 \left( \frac{k_{rf}eV_o}{E} \sin \Delta \psi \right)^2 \)

\( \sigma_z > \sqrt{\frac{\epsilon}{\beta_{TCAV}}} \left( \frac{\lambda_{rf}E}{2\pi eV_o} \right) \)

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Transverse deflector (TCAV)

1- calibration at zero crossing : $1^\circ RF$ to pixel

$1^\circ RF [S - Band] = 1ps$

$1^\circ RF [X - Band] = 250 fs$
Transverse deflector (TCAV)

Deconvolve beam size contribution

\[ \varphi_{RF} = -90^\circ \]

no field

\[ \varphi_{RF} = +90^\circ \]

\[
\begin{align*}
\sigma_y &= 43.98 \pm 2.76 \, \mu m \\
\sigma_z &= 5.800 \pm 0.755 \, \mu m \\
\text{cal} &= 7.235 \pm 0.633 \, \mu m/\mu m
\end{align*}
\]
LCLS bunch length measurements

OTR2 135 MeV

135 MeV 1.10 mm rms

14 GeV

BC1 Design Compression

PR55 14 GeV

14 GeV ‘Max’ Compression

97 A

10.3 ps

520 A

1.7 ps

950 A

0.89 ps

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Transverse deflector (TCAV)

Resolution: \( \sigma_z > \sqrt{\frac{\gamma \varepsilon_n}{\beta_{TCAV}}} \left( \frac{\lambda_{rf}}{eV_0} \right) \left( \frac{m_0 c^2}{2\pi} \right) \)

<table>
<thead>
<tr>
<th>Resolution</th>
<th>Tech</th>
<th>P[MW] V[MV]</th>
<th>L[m]</th>
<th>E[MeV]</th>
<th>( \varepsilon ) [mm-mrad]/( \beta ) [m]</th>
<th>Loc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>250 fs</td>
<td>S-Band (1)</td>
<td>0.5/0.7</td>
<td>0.4</td>
<td>135</td>
<td>0.3/5</td>
<td>LCLS inj.</td>
</tr>
<tr>
<td>11 fs (3)</td>
<td>S-Band</td>
<td>?/15</td>
<td>2.44</td>
<td>5000</td>
<td>0.5/60</td>
<td>LCLS main</td>
</tr>
<tr>
<td>33 fs</td>
<td>X-Band (2)</td>
<td>1/?</td>
<td>0.13</td>
<td>100</td>
<td>1/11</td>
<td>XTA</td>
</tr>
<tr>
<td>1 fs</td>
<td>X-Band</td>
<td>40/?</td>
<td>2.0</td>
<td>4300</td>
<td></td>
<td>LCLS dump</td>
</tr>
<tr>
<td>3 fs</td>
<td>X-Band</td>
<td>40/?</td>
<td>2.0</td>
<td>14000</td>
<td></td>
<td>LCLS dump</td>
</tr>
</tbody>
</table>

(1) S-Band is 2.856 GHz
(2) X-Band is 11.424 GHz
(3) Measured on wire scanner

(*) pulse stealing on LCLS (1Hz for a 120Hz operation)

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Transverse deflector (TCAV)
Longitudinal Phase space

Destructive (but “stealing mode”)
Single Shot

- XTCAV streaks horizontally;
- Dipole bends vertically.

\[ e^{-} \quad \sigma_{z} \quad \text{Undulator} \quad \text{X-band RF deflector} \quad \text{RF ‘streak’} \quad \text{Dipole} \]

\[ \Delta \psi \approx 90^\circ \]

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Longitudinal Phase space

Initial Measurement of e-beam and X-ray temporal profiles at LCLS using X-Band transverse deflector

Data taken on June 4\textsuperscript{th}, 2013. Beam energy 3.5GeV, 150pC. Temporal resolution is about 1.7fs rms in this test.

Preliminary results.
Longitudinal Phase space end LCLS injector

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... and all the students
Topics of interest

• Tomography
• Optical Replica
• Synchronization
Electro-Optic Sampling

T. Maxwell


B. Steffen

http://fla.desy.de/projects/eo/index_eng.html
http://www-library.desy.de/cgi-bin/showprep.pl?thesis07-020

Casalbuoni

http://prst-ab.aps.org/pdf/PRSTAB/v11/i7/e072802
Synchronization (AOS-based)

Cavalieri

http://prl.aps.org/abstract/PRL/v94/i11/e114801